UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 7
Power of a test.

Provide your **final answer only** for the following problems.

Problem 7.1. As the sample size increases, the power of a test will increase. *True or false?*

Solution: TRUE

Problem 7.2. You perform 2000 significance tests using a significance level 0.10. Under the assumption that all of the null hypotheses for the 2000 significance tests are true, how many of the 2000 significance tests would you expect to **not** result in a Type I error?
a. 200
b. 1800
c. 2000
d. 0
e. None of the above.

Solution: b.

Problem 7.3. A medical researcher is working on a new treatment for a certain type of cancer. The average survival time after diagnosis on the standard treatment is 2 years. In an early trial, she tries the new treatment on three subjects who have an average survival time after diagnosis of 4 years. Although the survival time has doubled, the results are not statistically significant, even at the 0.10 significance level.

Suppose, in fact, that the new treatment **does** increase the mean survival time in the population of all patients with this particular type of cancer. Which of the following statements is TRUE?
a. A Type I error occurred.
b. A Type II error occurred.
c. No error occurred.

Solution: b.

Problem 7.4. An engineer has designed an improved light bulb. The previous design had a mean lifetime of 1200 hours. Using a sample of 2000 of the new bulbs, the sample average lifetime of this improved light bulb is found to be 1201 hours. Although the difference is quite small, the effect was statistically significant at the 0.05 level. Suppose that, in fact, there is no difference between the mean lifetimes of the previous design and the new design. Which of the following statements is TRUE?
a. A Type I error occurred.
b. A Type II error occurred.
c. No error occurred.

Solution: a.

Provide your **complete solution** for the following problems.

Problem 7.5. The time needed for college students to complete a certain mirror-symmetry puzzle is modeled using a normal distribution with a mean of 30 seconds and a standard deviation of 3 seconds. You wish to see if the population mean time μ is changed by vigorous exercise, so you have a group of nine college students exercise vigorously for 30 minutes and then complete the puzzle.

i. What are your null and alternative hypotheses?

Solution: $H_0 : \mu = 30$ vs. $H_a : \mu \neq 30$

ii. What is the rejection region at the significance level 0.01?

Solution: The $z-$values corresponding to the two-sided hypothesis test at the 0.01 significance level are $z^* = \pm 2.576$. So, the rejection region is the complement of the interval

$$\left(30 - 2.576 \times \frac{3}{\sqrt{9}}, 30 + 2.576 \times \frac{3}{\sqrt{9}} \right) = (27.424, 32.576).$$

iii. What is the power of your test at $\mu = 28$ seconds?

Solution: We are looking for the probability of the event

$$27.424 < \bar{X}_n < 32.576$$

when

$$\bar{X}_n \sim N(mean = 28, variance = 1).$$

In standard, units, we are looking for the probability that

$$27.424 - 28 < Z < 32.576 - 28.$$

We get

$$\Phi(4.576) - \Phi(-0.576) = 1 - (1 - \Phi(0.58)) = \Phi(0.58) = 0.7190$$

Hence, the power of the test is $1 - 0.7190 = 0.2810$