UNIVERSITY OF TEXAS AT AUSTIN

Problem Set # 10

One-sample proportion inference.

Problem 10.1. A simple random sample of 100 bags of tortilla chips produced by Company X is selected every hour for quality control. In the current sample, 18 bags had more chips (measured in weight) than the labeled quantity. The quality control inspector wishes to use this information to calculate a 90% confidence interval for the true proportion of bags of tortilla chips that contain more than the label states. What is the value of the standard error of $\hat{p}$?

Solution:

$$\sqrt{\frac{0.18 \times 0.82}{100}} = \frac{0.3842}{10} = 0.03842.$$

Problem 10.2. A simple random sample of 60 blood donors is taken to estimate the proportion of donors with type A blood with a 95% confidence interval. In the sample, there are 10 people with type A blood. What is the margin of error for this confidence interval?

Solution:
The point estimate of the proportion of type-A blood donors in the population is 1/6 (according to the sample). The z^* associated with the 95% confidence interval is 1.96. So, the margin of error equals

$$1.96 \times \frac{\sqrt{(1/6) * (5/6)}}{\sqrt{60}} = 1.96 \times \frac{\sqrt{5}}{6\sqrt{60}} = 0.0943.$$

Problem 10.3. A simple random sample of 85 students is taken from a large university on the West Coast to estimate the proportion of students whose parents bought a car for them when they left for college. When interviewed, 51 students in the sample responded that their parents bought them a car. What is a 95% confidence interval for p, the population proportion of students whose parents bought a car for them when they left for college?

Solution:
The point estimate of the proportion of type-A blood donors in the population is 51/85 (according to the sample). The z^* associated with the 95% confidence interval is 1.96. So, the margin of error equals

$$1.96 \times \frac{\sqrt{(51/85) * (34/85)}}{\sqrt{85}} = 1.96 \times \frac{\sqrt{51 \times 34}}{85\sqrt{85}} = 0.1041.$$

Finally, the required confidence interval is

$$p = \hat{p} \pm z^* \times \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}} = 0.6 \pm 0.1041.$$

Problem 10.4. A simple random sample of 450 residents in the state of New York is taken to estimate the proportion of people who live within 1 mile of a hazardous waste site. It was found that 135 of the residents in the sample live within 1 mile of a hazardous waste site.

(1) What are the values of the sample proportion of people who live within 1 mile of a hazardous waste site and its standard error?
Solution:
$\hat{p} = 0.3$, and standard error $\sqrt{0.3 \times 0.7/450} = 0.0216$.

(2) What are the values of the sample proportion of people who live outside of the 1 mile radius around a hazardous waste site and its standard error?
Solution:
$\hat{p} = 0.7$, and standard error $\sqrt{0.3 \times 0.7/450} = 0.0216$.

(3) Do you notice something interesting about the above?

Solution: The standard errors are equal, so the width of the confidence interval would be the same in both cases.

Problem 10.5. **Sample size**

The *Information Technology Department* at a large university wishes to estimate the proportion p of students living in the dormitories who own a computer. They want to costruct a 90% confidence interval. What is the minimum required sample size the IT Department should use to estimate the proportion p with a margin of error no larger than 2 percentage points?

Solution:

$$n \geq \frac{(1.645)^2 \times 0.25}{(0.02)^2} = 1,691.27$$

Hence, the minimal sample size is $1,692$.

Problem 10.6. (5 points)
You want to design a study to estimate the proportion of people who strongly oppose to have a state lottery. You will use a 99% confidence interval and you would like the margin of error of the interval to be 0.05 or less. What is the minimal sample size required?

a. 666
b. 543
c. 385
d. Not enough information is provided.
e. None of the above.

Solution: a.