University of Texas at Austin

Problem Set # 3

The normal distribution.

Problem 3.1. The individual students' scores in the ACT exam are modeled using the normal distribution with an unknown mean (say, it varies from year to year) and with the **known** standard deviation of 6.

You take a SRS of students who took the ACT this year. The intention is to use their sample average to estimate (infer) the population mean.

You want the standard deviation of your statistic $\bar{X}_n$ to be at most 0.10. What is the least number of students you need to sample?

Solution: We are given that the standard deviation of the population distribution X is $\sigma_X = 6$. So, the standard deviation of the sample mean equals

$$SD[\bar{X}_n] = \frac{\sigma_X}{\sqrt{n}} = \frac{6}{\sqrt{n}}\,.$$

We want $SD[\bar{X}_n] \leq 0.1$. So, we need to have

$$\frac{6}{\sqrt{n}} \leq 0.1 \quad \Rightarrow \quad n \geq 3,600.$$

Problem 3.2. (5 pts)You are fiven a TRUE/FALSE exam with 30 questions. Suppose that you need to answer 21 questions correctly in order to pass. You have no idea what the class is about and decide to toss a fair coin to answer all the questions; you circle TRUE if the outcome is tails and you circle FALSE if the outcome is heads. What is your estimate of the probability p that you manage to pass the exam using this strategy?

Hint: It is best to use the **normal approximation** to get the approximate probability.

(a) $p \leq 0.0005$

(b) $0.0005 < p \leq 0.006$

(c) $0.006 < p \leq 0.04$

(d) $0.04 < p$

(e) None of the above

Solution: (c)

Let us denote the number of correct answers you get using the coin-toss strategy by X. Then, $X \sim b(30, 1/2)$. The mean of X is $30 \cdot \frac{1}{2} = 15$ and its variance is $30 \cdot \frac{1}{2} \cdot \frac{1}{2} = 7.5$. So, the standard deviation of X is $\sqrt{7.5} \approx 2.74$. We can express the probability p as

$$p = \mathbb{P}[X \geq 21] = \mathbb{P}[X \geq 20.5] = \mathbb{P}[\frac{X-15}{2.74} \geq \frac{20.5-15}{2.74}] \approx \mathbb{P}[\frac{X-15}{2.74} \geq 2].$$

This probability is approximately $\Phi(+\infty) - \Phi(2) = 1 - \Phi(2) = 1 - 0.97725 = 0.02275$.