University of Texas at Austin

Problem Set # 2

Causation. Producing data.

Problem 2.1. (5 points) *What is wrong about the following report?*

An experiment is conducted to study the bonding strength of adhesives that contain varying amounts of a particular chemical additive. Wafers of a specified material are glued together using the adhesive with each amount of additive, allowed to set for 24 hours, and then the strength needed to separate the wafers is determined. It is reported that the correlation between strength required and amount of additive was 0.86 pounds-force per square inch.

Solution: The correlation is by definition **unitless**.

Problem 2.2. (2 points)
Some observed associations between two variables are due to a lurking variable rather than a cause-and-effect relationship between the two variables.
True or false?

Solution: **TRUE**

Problem 2.3. (2 points)
Two variables are confounded when their effects on a response variable can be distinguished from each other.
True or false?

Solution: **FALSE**

Problem 2.4. (5 points) To examine the relationship between two variables, the variables must be measured from the same ...

a.: ... units.
b.: ... cases.
c.: ... values.
d.: ... parameters.
e.: None of the above is correct.

Solution: **(b)**

Problem 2.5. (5 points)
Midsomer University "flips" the classroom for a group of 250 students. Of these students, 150 receive an A in the end of the term. From this information you conclude ...

a.: ... "flipping" the classroom is an extremely effective way of teaching.
b.: ... nothing, because the sample size is too small.
c.: ... "flipping" the classroom should become priority on the public education front because it evidently works.
d.: ... nothing, because there is no control group.
e.: None of the above.

Solution: d.

Problem 2.6. Disneyland

There are four people in a family: a father, a mother and two children. They have won two tickets to go to Disneyland for a week. They decide to select a sample of two people for the trip as follows: The mother and father flip a coin to see which of the two of them will go, and they then flip a coin to see which of the two children will go. This is . . .

a. a *simple random sample* of size two from the family since two coins were flipped.

b. a *probability sample* from the family since each member of the family has a known chance of being selected to go on the trip.

c. not a *probability sample* since the mother and father can't go together.

Solution: b.

A probability sample just gives each member of the population a known chance of being selected, in this case each person has a 50% chance to go (see the explanation for answer choice (b)). The fact that the mother and father can't go together just tells you that not all samples have the same chance, which means that it is not a simple random sample of two people from the family.

Problem 2.7. (5 points) **French "unfriendliness"**

A study sponsored by American Express Co. and the French government tourist office found that old American stereotypes about French unfriendliness weren't true. The respondents were more than 1000 Americans who have visited France more than once for pleasure over the past two years. The results of this study are probably . . .

a. very accurate given the large sample size.

b. very inaccurate since the sample is only a small fraction of all Americans who have visited France.

c. biased, overstating the extent to which the old stereotypes weren't true.

Solution: c.

The sample is not a random sample of Americans. Americans who visited France more than once for pleasure over the past two years may be more likely to find the French friendly than other Americans, and this will overstate the extent to which the old stereotypes are not true. This is because those who had a negative experience on their first visit are less likely to return.

Problem 2.8. (8 points)

The *Didactic Digest* recently published the results of a pair of studies aimed at exploring the effects of listening to **Wolfgang Amadeus Mozart** versus **Kanye West** while studying. All the students in either experimental design were given identical materials they were allowed to study followed by a short exam on the studied material.

The two studies were designed so that:

Study I allowed the students to decide whether to study either listening to **Mozart**, or to **Kanye**;

Study II randomly assigned the students to study either listening to **Mozart**, or to **Kanye**.

In *Study I*, it was discovered that the Mozart listeners scored **significantly better** than the Kanye listeners. *Study II* did not uncover any signifficant difference between the two groups of students.

The overall exam average in both studies was roughly the same.

i. (2 points) Which study had controls?
Solution: Both did.

ii. (2 points) Which study had randomized controls?
Solution: *Study II* only.

iii. (2 points) Which study is more likely to have confounders?
Solution: *Study I*.

iv. (2 points) Which of the following claims is best supported by the above results?
(1) Students learn better if they get to choose their own music.
(2) Students learn better if thay listen to music.
(3) Classical music appears to enhance learning.
(4) Students who chose **Mozart** are different in more ways than just their musical tastes.
Solution: Only the last claim can in any way be supported by the studies' findings.

Problem 2.9. (5 points) A medical researcher thinks that adding calcium to the diet will help reduce blood pressure. She believes that the effect is different for men and women. 20 men and 20 women are willing to participate in the study. The researcher chooses 10 of the men and 10 of the women at random. These chosen 20 men and women take a calcium pill every day. The other 20 men and women take a placebo. This is a . . .

- **a.:** stratified random sample design.
- **b.:** simple random sample design.
- **c.:** randomized block experimental design.
- **d.:** completely randomized experimental design.
- **e.:** None of the above is correct.

Solution: c.

Problem 2.10. How to get an answer without asking the question?
The College of Education wanted to gauge the percentage of cheaters in the student population at Wobegon University. They were inspired by the last line in their motto:

"Where all the women are strong,
All the men are good looking,
and all the children are above average.

Realizing the obvious problems with conducting a survey which outright asks the questions: "Are you or have you ever been a cheater?", they devised the following ruse:

A huge bag was filled with many, many question slips. On exactly 70% of the slips, the question

"Have you ever cheated on an exam?"

was written. On the remaining question slips, the question

"Is the last digit of your social security number even?"

was written. Each subject randomly drew a question slip from the bag, read the question in a clandestine manner, responded to the interviewer with a "yes" or "no", and burnt the question slip at the ritual bonfire provided for this occasion. Of course, the interviewer did not know the actual question asked, just the ultimate response. So, there was no real reason for the subject to lie, and we assume that the subjects responded truthfully.

Assume that precisely 50% of the population has an even last digit of the social security number. Let's first formalize the given information in the language of probability.

Solution: Define the events

$$C = \{\text{the subject got the "cheater question slip"}\},$$
$$D = \{\text{the subject's response was "yes"}\}.$$

So,

$$C^c = \{\text{the subject got the "SSN question slip"}\}.$$

With this notation, the information from the problem can be formalized as

$$\mathbb{P}[C] = 0.7, \quad \mathbb{P}[D \mid C^c] = 0.5.$$

Question #1. It turned out that 44% of the subjects answered "yes". Give an estimate of the proportion of *cheaters* in this population.

Solution: Now, we are given that $\mathbb{P}[D] = 0.44$. Our goal is, however, to figure out

$$\mathbb{P}[D \mid C] = ???$$

By the *Law of Total Probability*,

$$\mathbb{P}[D] = \mathbb{P}[D \cap C] + \mathbb{P}[D \cap C^c] = \mathbb{P}[D \mid C]\mathbb{P}[C] + \mathbb{P}[D \mid C^c]\mathbb{P}[C^c]$$

So,

$$0.7\mathbb{P}[D \mid C] = 0.44 - 0.5 \times 0.3 = 0.29 \quad \Rightarrow \quad \mathbb{P}[D \mid C] = 0.4143.$$

Question #2. What percentage of "yes" answers would you have obtained had *all* the subjects in the population been *cheaters*?

Solution: Now, we are given that $\mathbb{P}[D \mid C] = 1$. So, using the same technique as above, i.e., the *Law of Total Probability*, we get

$$\begin{aligned}\mathbb{P}[D] &= \mathbb{P}[D \cap C] + \mathbb{P}[D \cap C^c] \\ &= \mathbb{P}[D \mid C]\mathbb{P}[C] + \mathbb{P}[D \mid C^c]\mathbb{P}[C^c] \\ &= 1 \times 0.7 + 0.5 \times 0.3 = 0.7 + 0.15 = 0.85.\end{aligned}$$

Problem 2.11. (13 points) **Mirrored randomized response**

People text while driving, but they are reluctant to admit having done so. You devise an experiment that would help you estimate the proportion of these reckless individuals in the population.

The experiment is conducted in the following way: The subject spins a spinner which lands either on *Question A*, or on *Question B* with the questions being as follows:

Question A: " I have never, not even once, texted while driving."

Question B: "I have at least once texted while driving."

Getting *Question A* is twice as likely as getting *Question B*. The interviewer does not know the actual question asked, just the ultimate response. So, there was no real reason for the subject to lie, and we assume that the subjects responded truthfully.

i. (8 points)

It turned out that 50% of the subjects answered "yes". Give an estimate of the proportion of *terrible texter-drivers* in this population.

Solution:

The probability of getting *Question A* is 2/3 while the probability of getting *Question B* is 1/3. We are given that $\mathbb{P}[D] = 0.50$ with the event D defined as

$$D = \{\text{the subject answered } \textbf{Yes}\}.$$

Our goal is to figure out p such that (by the *Law of Total Probability*),

$$1/2 = \mathbb{P}[D] = (2/3)(1-p) + (1/3)p \quad \Rightarrow \quad p = 1/2.$$

ii. (5 points) What percentage of "yes" answers would you have obtained in an ideal world in which nobody ever endangers the public by simultaneously texting and driving?

Solution:

$$(2/3)1 + (1/3)0 = 2/3.$$