Hermitian metrics

Def X almost cplx:

A Riem. metric g on X is <u>Hermitian</u> if $\forall x \in X$ g_x is compatible with I_x.

The induced $\omega \in \Lambda^2 T^* X$ is called the <u>fundamental form</u> of g.

If X cplx, then locally a Hermitian metric looks like

$$g = \frac{1}{2} \sum_{i,j} h_{ij}(z, \bar{z})(dz_i \otimes d\bar{z}_j + d\bar{z}_j \otimes dz_i) \qquad \left[h_{ij} + \text{def. Hermitian} \right]$$

and

$$\omega = \frac{i}{2} \sum_{i,j} h_{ij} \, dz_i \wedge d\bar{z}_j \qquad \left[\text{factor of } i \text{ makes } \omega \text{ real} \right]$$

Notation $\Lambda^k X = \Lambda^k T^* X$

Def X almost cplx with Hermitian metric

$\Rightarrow$ fibrewise linear algebra gives homomorphisms of vector bundles,

$$L : \Lambda^k X \longrightarrow \Lambda^{k+2} X \qquad \Lambda : \Lambda^k X \longrightarrow \Lambda^{k-2} X$$
$$* : \Lambda^k X \longrightarrow \Lambda^{2n-k} X \qquad H : \Lambda^k X \longrightarrow \Lambda^k X$$

Def $P^k X = \mathrm{Ker}(\Lambda : \Lambda^k X \longrightarrow \Lambda^{k-2} X)$

Cor $\Lambda^k X = \bigoplus_{i \geq 0} L^i (P^{k-2i} X)$