Lecture 5

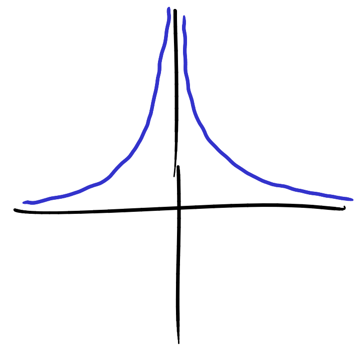
10 Sep 2018

My office hours: M 2-3 ← today
(RLM 9.134) W 4-5

Last time: limits $\lim_{x\to a} f(x) = L$

$\lim_{x\to a} f(x) = \infty$

eg $\lim_{x\to 0} \frac{1}{x^2} = \infty$



Q 1) What is $\lim_{x\to -3} \frac{x}{(x+3)^2}$?

$-\infty$, DNE, 0

plug in $x=-3$: $\frac{-3}{(3-3)^2} = \frac{-3}{0} \to$ no help

if x is close to -3:

$\frac{x}{(x+3)^2} = \frac{(\text{close to } -3)}{(\text{very small positive})}$

$= (\text{very large negative})$

So $\lim_{x\to -3} \frac{x}{(x+3)^2} = \underline{\underline{-\infty}}$

2) What is $\lim_{x\to 2} \frac{x-2}{x^2-4}$?

plug in $x=2$: $\frac{2-2}{4-4} = \frac{0}{0} \longrightarrow$ no help

but! $\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2} \frac{x-2}{(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{1}{x+2} = \underline{\underline{\frac{1}{4}}}$

Ex $\lim_{x\to 1} \frac{1}{x-1}$

if x is slightly bigger than 1 $\quad x = 1.00001$

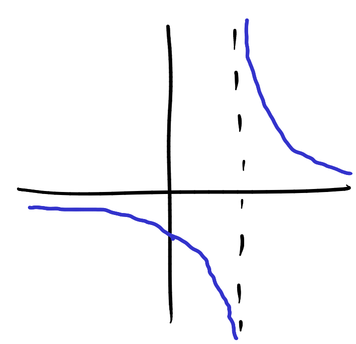
then $\frac{1}{x-1}$ = big positive

if x slightly smaller than 1 $\quad x = 0.99999$

then $\frac{1}{x-1}$ = big negative

So,

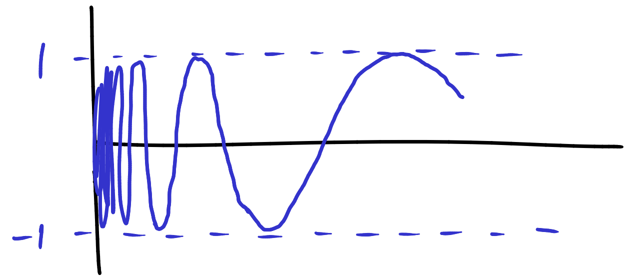
$\lim_{x\to 1^+} \frac{1}{x-1} = +\infty \qquad \lim_{x\to 1^-} \frac{1}{x-1} = -\infty$



$$\lim_{x\to 1} \frac{1}{x-1} \text{ DNE.}$$

<u>Ex</u> $\lim_{x\to 0^+} \sin\left(\frac{1}{x}\right)$

DNE.



<u>Limit Laws</u>

Suppose c is any constant, and the limits $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist.

Then:

1. $\lim_{x\to a} (f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a} (f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a} c\cdot f(x) = c\cdot \lim_{x\to a} f(x)$
4. $\lim_{x\to a} f(x)\cdot g(x) = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right).$

   <u>Ex</u> if $\lim_{x\to 3} f(x) = 7$, $\lim_{x\to 3} g(x) = 8$ then $\lim_{x\to 3} f(x)g(x) = 56.$
5. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ (<u>if</u> $\lim_{x\to a} g(x) \neq 0$!)
6. $\lim_{x\to a} f(x)^n = \left(\lim_{x\to a} f(x)\right)^n$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

<u>Ex</u>
$$\lim_{x\to 0} \frac{x^2}{3\sin^2 x} = \frac{1}{3}\lim_{x\to 0} \frac{x^2}{\sin^2 x}$$
$$= \frac{1}{3}\left(\lim_{x\to 0} \frac{x}{\sin x}\right)^2$$
$$= \frac{1}{3}\left(\frac{1}{\lim_{x\to 0} \frac{\sin x}{x}}\right)^2$$
$$= \frac{1}{3}\left(\frac{1}{1}\right)^2 = \underline{\underline{\frac{1}{3}}}$$

or.
$$\frac{1}{3}\left(\lim_{x\to 0} \frac{\sin x}{x}\right)^{-2}$$
$$= \frac{1}{3}\, 1^{-2}$$
$$= \frac{1}{3}$$

⑦ $\lim_{x\to a} c = c$ — Ex $\lim_{x\to 7} \sqrt{2} = \sqrt{2}$

⑧ $\lim_{x\to a} x = a$ — Ex $\lim_{x\to 7} x = 7$

⑨ $\lim_{x\to a} x^n = a^n$

⑩ $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$

⑪ $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

Q $\lim_{x\to 0} x^2+9 = ?$

$= 9$

$\lim_{x\to 0} \sqrt{x^2+9} = ?$

$= \sqrt{\lim_{x\to 0} x^2+9}$ (by Limit Law 11)

$= \sqrt{\lim_{x\to 0} x^2 + \lim_{x\to 0} 9}$

$= \sqrt{0^2+9}$

$= \sqrt{9} = \underline{\underline{3}}$

Ex $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2} = ?$

plug in 0: $\frac{3-3}{0^2} = \frac{0}{0}$ no help

simplify: $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2} \cdot \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3} = \lim_{x\to 0} \frac{x^2+9-9}{x^2(\sqrt{x^2+9}+3)}$

$= \lim_{x\to 0} \frac{x^2}{x^2(\sqrt{x^2+9}+3)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+9}+3} = \underline{\underline{\frac{1}{6}}}$