Lecture 7 14 Sep 2018

Last time: limit laws and continuity

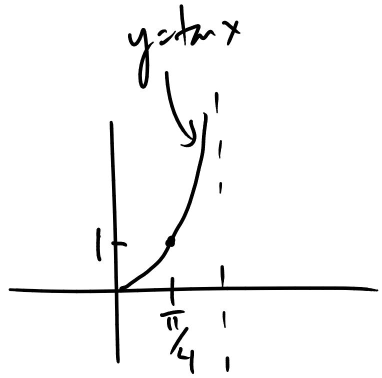
Ex $\lim_{x\to 0} \tan\left(\frac{\pi}{4}\cdot\frac{\sin^2 x}{x^2}\right)$

$\overset{?}{=} \tan\left(\frac{\pi}{4}\cdot \lim_{x\to 0}\frac{\sin^2 x}{x^2}\right)$

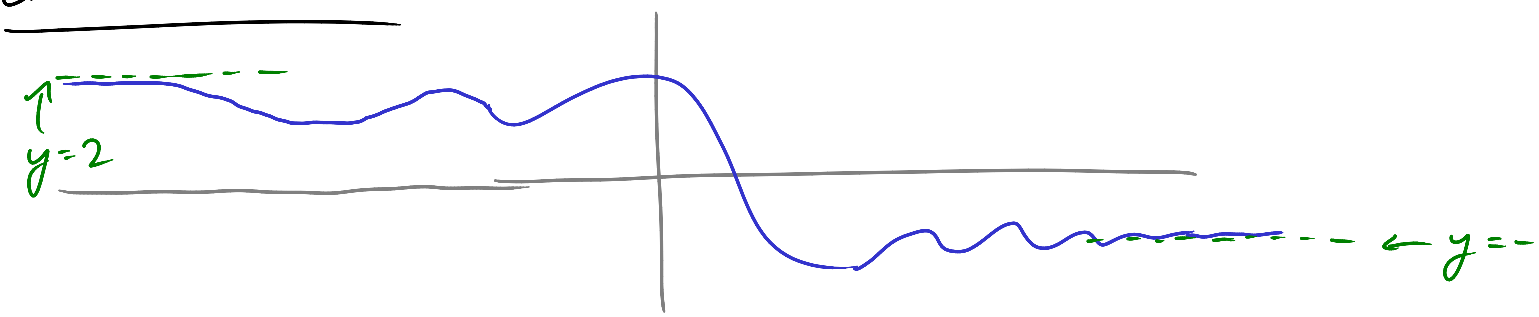
OK, because tan is continuous at $\pi/4$

$= \tan\left(\frac{\pi}{4}\cdot 1\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$ ✓



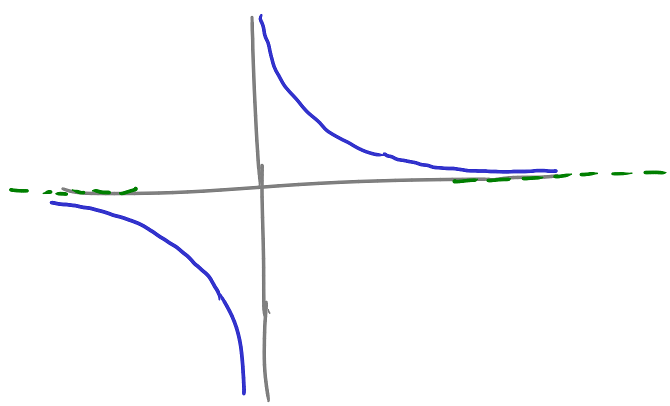
Limits as $x \to \pm\infty$



$\lim_{x\to\infty} f(x) = -1$ $\lim_{x\to -\infty} f(x) = 2$ or: graph of $y=f(x)$ has horiz asymptotes at $y=2$, $y=-1$

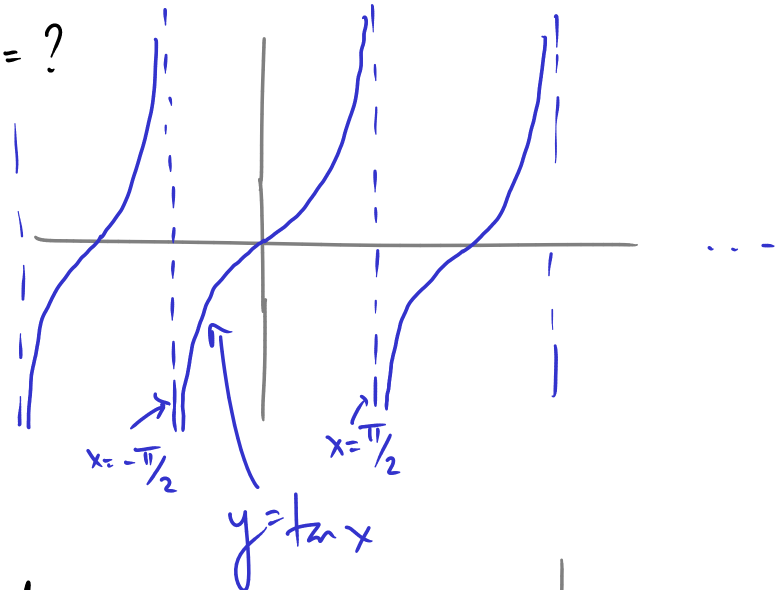
Ex $\lim_{x\to\infty}\frac{1}{x} = 0.$

$\lim_{x\to -\infty}\frac{1}{x} = 0.$

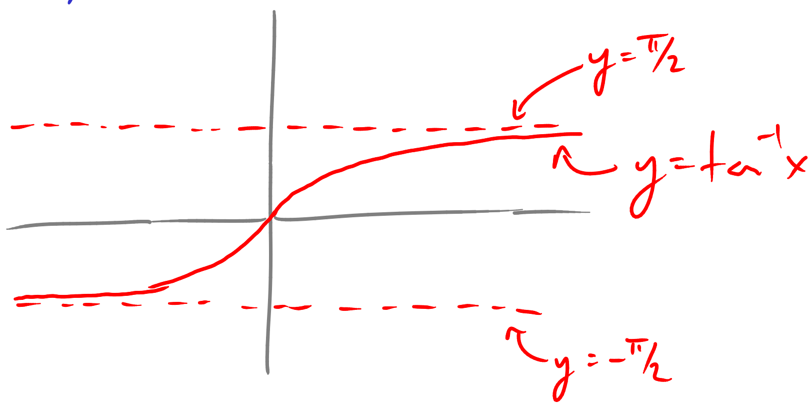


Ex $\lim_{x\to\infty} 6 + \frac{1}{x^2}$ (plug in very large positive x, then $6+\frac{1}{x^2} = 6 + \frac{1}{(\text{very big})} = 6 + (\text{very small}) \to \underline{6}$

Ex $\lim_{x\to\infty} \tan^{-1} x = ?$



reflect this graph in line $y=x$ to get:

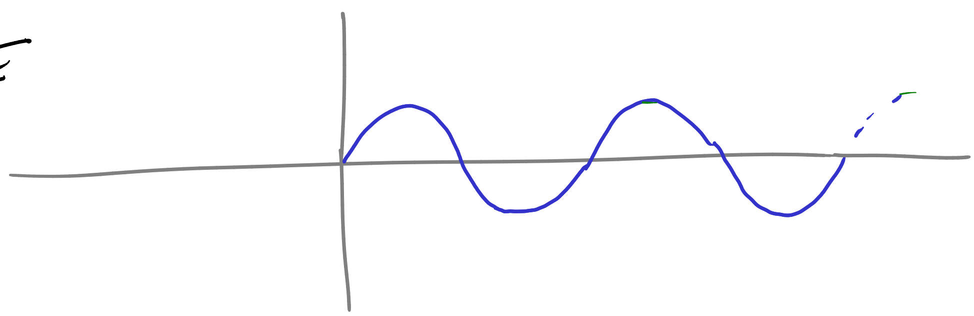


$$\lim_{x\to\infty} \tan^{-1} x = \underline{\underline{\frac{\pi}{2}}}$$

$$\lim_{x\to-\infty} \tan^{-1} x = -\frac{\pi}{2}$$

Ex $\lim_{x\to\infty} \sin x = \text{DNE}$

(graph doesn't approach any horiz asymptote as $x \to \infty$)



Fact Can use Limit Laws for limits as $x\to\infty$ or $x\to-\infty$ just as we do for $x\to a$.

Ex $\lim_{x\to\infty} 18+\frac{1}{x^{4/3}} = \left(\lim_{x\to\infty} 18\right) + \left(\lim_{x\to\infty} \frac{1}{x^{4/3}}\right)$

$$= 18+0 = \underline{\underline{18}}$$

Ex $\lim_{x\to\infty} \frac{x^2+4x+7}{x^2-9} = ?$

plug in very large x: get $\frac{(\text{big})}{(\text{big})}$ $\frac{\infty}{\infty}$

— not very helpful.

more quantitative: if $x = 1000$,

$$\frac{1000^2+4000+7}{1000^2-9}$$

$$= \frac{1000000+4007}{1000000-9} \approx 1$$

— looks like only the biggest power of x in num. and denom matters!

ie $\lim_{x\to\infty} \frac{x^2+4x+7}{x^2-9} = \lim_{x\to\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} 1 = 1.$

To prove it:

$$\frac{x^2+4x+7}{x^2-9} = \frac{(x^2+4x+7)\frac{1}{x^2}}{(x^2-9)\cdot\frac{1}{x^2}} = \frac{1+\frac{4}{x}+\frac{7}{x^2}}{1-\frac{9}{x^2}}$$

So $\lim_{x\to\infty} \frac{x^2+4x+7}{x^2-9} = \lim_{x\to\infty} \frac{1+\frac{4}{x}+\frac{7}{x^2}}{1-\frac{9}{x^2}} = \frac{\lim_{x\to\infty} 1+\frac{4}{x}+\frac{7}{x^2}}{\lim_{x\to\infty} 1-\frac{9}{x^2}} = \frac{1}{1} = \underline{1}$

Q $\lim_{x\to 4} \frac{\sin x}{x} = \frac{\sin 4}{4}$ $\qquad \lim_{x\to 0} \frac{\sin x}{x} = 1$

Ex $\lim_{x\to\infty} \frac{9x^3-8x}{x^3+7x+8} = \underline{9}$ $\qquad \lim_{x\to\infty} \frac{x+3}{74+8x} = \underline{\frac{1}{8}}$

$\lim_{x\to\infty} \frac{1+2x}{x^2} = \underline{0}$

Ex $\lim_{x\to\infty} \sqrt{x^2+1} - x$

plug in large x: $\sqrt{(\text{big})^2+1} - (\text{big})$

$= (\text{big}) - (\text{big})$ no help

" $\infty - \infty$ "

$$= \lim_{x \to \infty} \left(\sqrt{x^2+1} - x\right) \cdot \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x}$$

$$= \lim_{x \to \infty} \frac{(x^2+1) - x^2}{\sqrt{x^2+1} + x}$$

$$= \lim_{x \to \infty} \frac{1}{x + \sqrt{x^2+1}} = \underline{\underline{0}} \qquad \left(\frac{1}{\infty}\right)$$

$$\lim_{x \to \infty} \sqrt{x^2+1} + x = \underline{\underline{\infty}}$$

$$\text{"}\infty + \infty\text{"}$$

$$(x^2+1)^{\frac{1}{2}} \neq (x^2)^{\frac{1}{2}} + 1^{\frac{1}{2}}$$

$$(a+b)^2 \neq a^2+b^2$$

$$\lim_{x \to -\infty} \frac{x+3}{\sqrt{9x^2-5x}} = \lim_{x \to -\infty} \frac{x+3}{\sqrt{9x^2}} = \lim_{x \to -\infty} \frac{x+3}{|3x|}$$

$$= \lim_{x \to -\infty} \frac{x}{|3x|}$$

$$= \lim_{x \to -\infty} \left(\frac{x}{-3x}\right) = \underline{\underline{-\frac{1}{3}}}$$

$$\sqrt{x^2} = |x|$$

$$\sqrt{9x^2} = 3|x|$$