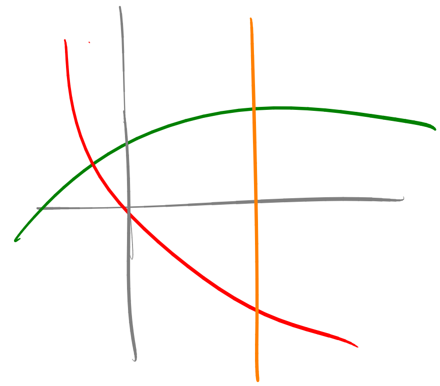
Lecture 20 | 17 Oct 2012

Housekeeping: my wife's baby due 23 Oct

Last time: double integrals over general regions



Sequences (Ch 11.1)

A sequence is an ordered list of numbers

$$\{a_n\} = a_1, a_2, a_3, a_4, \dots, a_{100}, \dots$$

Ex

$a_n = n$: $1, 2, 3, 4, 5, \dots, 100, \dots$

$\uparrow \; a_1, a_2, a_3, \dots, a_{100}, \dots$

$a_n = (-1)^n$: $-1, 1, -1, 1, -1, \dots, 1, \dots$

$\uparrow \; a_1, a_2, a_3, \dots, a_{100}, \dots$

$a_n = \dfrac{1}{n^2+1}$: $\dfrac{1}{2}, \dfrac{1}{5}, \dfrac{1}{10}, \dots, \dfrac{1}{10001}, \dots$

a_n = the closing price of DJIA on the n^{th} trading day of this year:

10784.6, 10812.9, 10800.3, ...

$a_n =$ the n^{th} term of the Fibonacci sequence (i.e. $a_1 = 1$, $a_2 = 1$, $a_{n+2} = a_n + a_{n+1}$)

$$1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

a_1, a_2, $a_3 = a_1 + a_2$, $a_4 = a_2 + a_3$

$$a_n = \frac{1}{n}\cos\left(\frac{n\pi}{2}\right)$$

$$0, -\frac{1}{2}, 0, \frac{1}{4}, 0, -\frac{1}{6}, 0, \frac{1}{8}, 0, -\frac{1}{10}, 0, \frac{1}{12}, 0, -\frac{1}{14}, \ldots$$

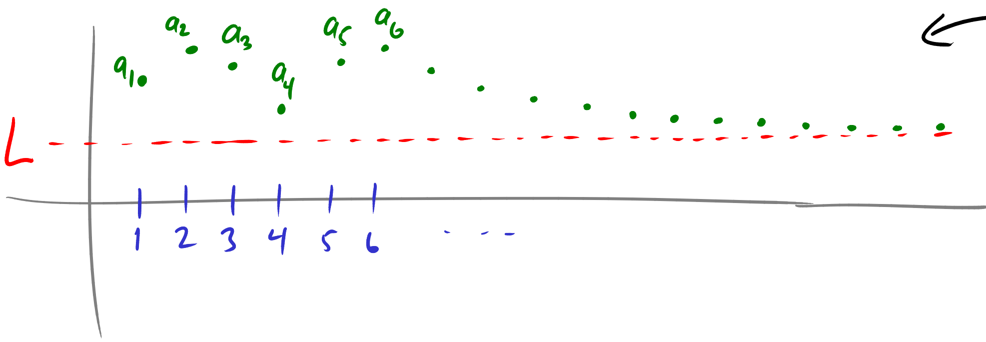
Ex Consider the sequence $3, 8, 13, 18, 23, 28, \ldots$ where each term differs from the previous one by 5. What is a_n?

$$a_n = 5n - 2$$

Ex $a_n = n!$ $\left[n! = (n)(n-1)(n-2)\cdots(1) \quad \text{e.g. } 4! = 4\cdot 3\cdot 2\cdot 1 = 24 \right]$

$$1, 2, 6, 24, 120, 720, \ldots$$

Fundamental question about a sequence $\{a_n\}$: <u>does it converge</u>?

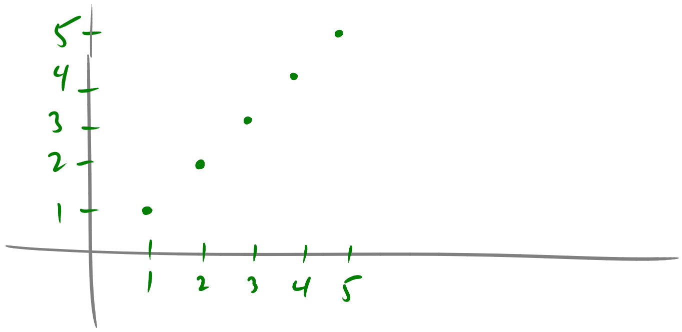


this sequence $\{a_n\}$ converges to L.

We say $\{a_n\}$ <u>converges to L</u> if by making n big enough we can make a_n as close to L as we like. (For more formal def, see p. 692 of text, "Def 2")

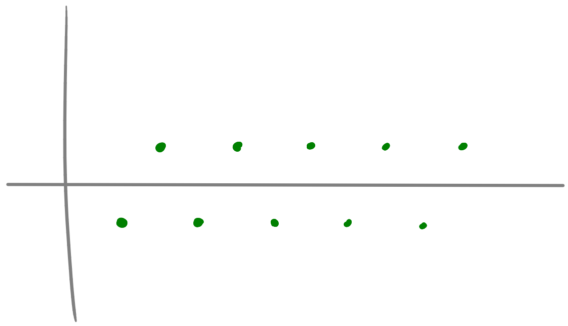
We say $\{a_n\}$ <u>diverges</u> if it does not converge to any L.

$a_n = n$:



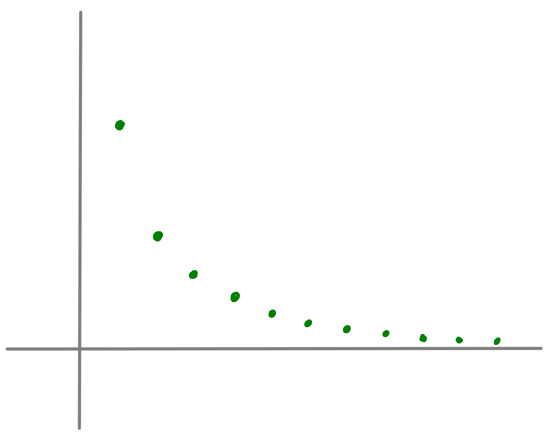
<u>diverges</u>

$a_n = (-1)^n$:



<u>diverges</u>

$a_n = \frac{1}{n}$:

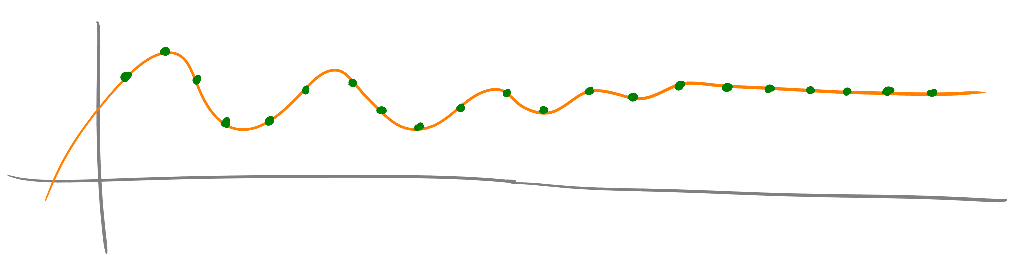


<u>converges</u> to 0

We also express this as $\lim\limits_{n\to\infty} a_n = 0$, i.e. $\lim\limits_{n\to\infty} \frac{1}{n} = 0$.

This might remind you of the fact that $\lim\limits_{x\to\infty} \frac{1}{x} = 0$.

Indeed: <u>if</u> our sequence is given by a <u>function</u>, $a_n = f(n)$, and $\lim\limits_{x\to\infty} f(x) = L$, then $\lim\limits_{n\to\infty} a_n = L$, i.e. $\{a_n\}$ converges to L.



Ex Does $a_n = \frac{\ln n}{n}$ converge?

Look at $f(x) = \frac{\ln x}{x}$. $\lim_{x\to\infty} \frac{\ln x}{x}$ "=" $\frac{\infty}{\infty}$, use L'H rule:

$$= \lim_{x\to\infty} \frac{(1/x)}{1} = \lim_{x\to\infty} \frac{1}{x} \text{ "=" } \frac{1}{\infty} = 0$$

So, $a_n = \frac{\ln n}{n}$ converges, to 0.

Ex Does $a_n = \frac{\ln n}{n} - 4$ converge, and to what?

$$\lim_{n\to\infty}\left(\frac{\ln n}{n} - 4\right) = \lim_{n\to\infty}\left(\frac{\ln n}{n}\right) - \lim_{n\to\infty}(4)$$

$$= 0 - 4 = -4$$

$\Rightarrow$ Yes, it converges to -4

This was an example of "Limit Laws for Sequences":

If a_n, b_n are 2 sequences which converge, then

$$\lim_{n\to\infty}(a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$$

$$\lim_{n\to\infty} c \cdot a_n = c \cdot \lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} a_n \cdot b_n = \left(\lim_{n\to\infty} a_n\right) \cdot \left(\lim_{n\to\infty} b_n\right)$$

$$\lim_{n\to\infty} a_n^k = \left(\lim_{n\to\infty} a_n\right)^k$$

Ex Does $a_n = \sin\left(\frac{\pi n}{1+4n}\right)$ converge?

Look at $f(x) = \sin\left(\frac{\pi x}{1+4x}\right)$

$$\lim_{x\to\infty} \sin\left(\frac{\pi x}{1+4x}\right) = \lim_{x\to\infty} \sin\left(\frac{\pi x \cdot \frac{1}{x}}{(1+4x)\cdot\frac{1}{x}}\right) = \lim_{x\to\infty} \sin\left(\frac{\pi}{4+1/x}\right)$$

$$= \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

So a_n conv. to $\frac{\sqrt{2}}{2}$