Lecture 39

Final exam: Fri Dec 14 7-10pm BUR 106

I will hold review session Tue Dec 11 12-3pm

Trigonometric Substitution

When should we consider trig sub?

Usually when we face $\int$ involving

$\sqrt{1-x^2}$ try $x = \sin\theta$ or $x = \cos\theta$

$\sqrt{x^2-1}$ try $x = \sec\theta$

$\sqrt{x^2+1}$ try $x = \tan\theta$

Ex
$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$$

Try $x = \sin\theta$: $dx = \cos\theta\, d\theta$ $\quad \sin^{-1}x = \sin^{-1}(\sin\theta) = \theta$

$$\int \frac{\theta}{\sqrt{1-\sin^2\theta}} \cos\theta\, d\theta = \int \frac{\theta}{\cos\theta} \cos\theta\, d\theta$$

$$= \int \theta\, d\theta = \frac{1}{2}\theta^2 + C$$

$\theta = \sin^{-1}x$ so $\displaystyle\int = \underline{\frac{1}{2}\left(\sin^{-1}x\right)^2 + C}$

But: We should only use trig-sub if all else fails!

Ex
$$\int \frac{7x}{\sqrt{1-x^2}}\, dx$$

$u = 1-x^2$

$du = -2x\, dx$

$-\frac{1}{2}du = x\, dx$

$$= \int \frac{7\left(-\frac{1}{2}du\right)}{\sqrt{u}} = -\frac{7}{2}\int \frac{du}{\sqrt{u}} = -\frac{7}{2}\int u^{-1/2}\,du$$

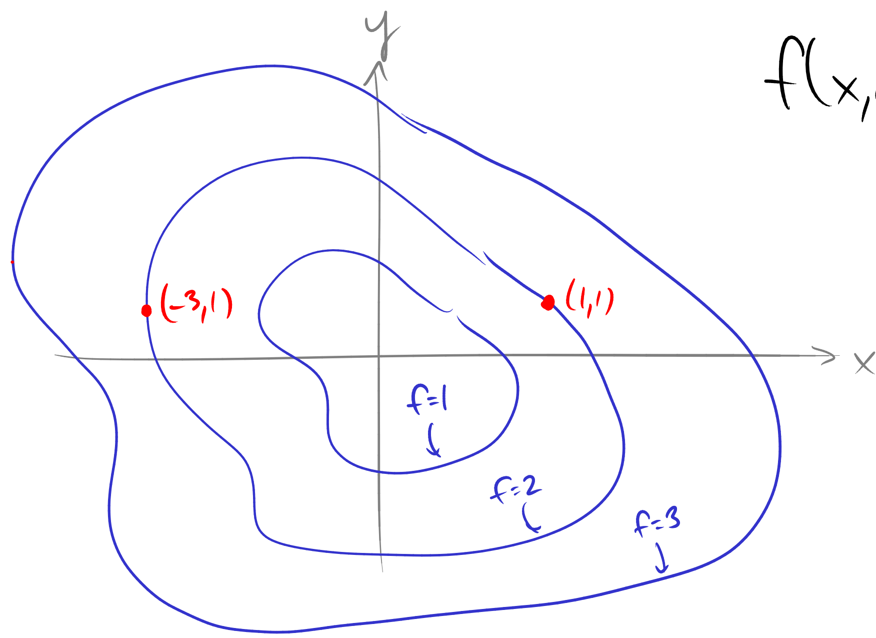
$$= -\frac{7}{2}\left(2u^{1/2}\right) + C$$

$$= -7u^{1/2} + C$$

$$= -7\sqrt{1-x^2} + C$$

Contour maps

$f(x,y)$



Is $f_x(1,1)$ positive or negative? **Positive**

$f_y(1,1)$ " " " ? **Positive**

Is $f_x(-3,1)$ + or − ? **Negative**

$f_y(-3,1)$ + or − ? **Zero**

If $g(x) = \int_{35}^{x^2} \tan u \, du$ what is $g'(x)$? $g'(\pi/4)$?

$g'(x) = 2x \tan x$ $\quad g'(\pi/4) = \pi/2$

Ex

$$\int 6x(\ln x)^2\,dx \qquad = \int u\,dv$$

IBP: $u = (\ln x)^2 \qquad v = 3x^2$

$du = 2\frac{\ln x}{x}\,dx \qquad dv = 6x\,dx$

$$= uv - \int v\,du$$

$$= 3x^2(\ln x)^2 - \int 3x^2 \cdot 2\frac{\ln x}{x}\,dx$$

$$= 3x^2(\ln x)^2 - \int 6x\ln x\,dx$$

then do IBP again: $u = \ln x \qquad v = 3x^2$

$du = \frac{1}{x}\,dx \qquad dv = 6x\,dx$

$$= 3x^2(\ln x)^2 - \left(uv - \int v\,du\right)$$

$$= 3x^2(\ln x)^2 - 3x^2(\ln x) + \int 3x^2 \cdot \frac{1}{x}\,dx$$

$$= \underline{3x^2(\ln x)^2 - 3x^2\ln x + \frac{3}{2}x^2 + C}$$

$$\int \frac{1+x^2}{1-x^2}\,dx$$

$$\begin{array}{r|l} & -1 \\ -x^2+1 & x^2+0x+1 \\ & x^2 \quad\;\; -1 \\ \hline & \qquad\quad 2 \end{array}$$

$$= \int -1 + \frac{2}{1-x^2}\,dx \qquad = \frac{-(1-x^2)+2}{1-x^2} = \frac{1+x^2}{1-x^2}$$

$$\frac{2}{1-x^2} = \frac{A}{1-x} + \frac{B}{1+x}$$

$$2 = A(1+x) + B(1-x)$$

plug in $x=1$: $2 = 2A$ so $A=1$

$x=-1$: $2 = 2B$ $B=1$

So $\int = \int -1 + \frac{1}{1-x} + \frac{1}{1+x}\,dx$

$$= -x - \ln(1-x) + \ln(1+x) + C$$

$$= -x + \ln\left(\frac{1+x}{1-x}\right) + C$$

$$\int \sin^3 x \cos^2 x\,dx$$

$$= \int \sin^2 x \cos^2 x\,(\sin x\,dx)$$

$u = \cos x$

$du = -\sin x\,dx$

$$= \int (1-\cos^2 x)\cos^2 x\,(\sin x\,dx)$$

$$= \int (1-u^2)\,u^2\,(-du)$$

$$\int \sin^2 x\,dx$$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$x=0$ ✓

$x=\pi/4$ ✓

$$= \int \frac{1}{2}(1-\cos 2x)\,dx$$

Find the Taylor series for $\ln x$ around $a=5$.

$x-5$

Trick: $\ln x = \ln(5+(x-5))$

$$= \ln\left(5\left(1+\frac{x-5}{5}\right)\right)$$

$$= \ln 5 + \ln\left(1+\frac{x-5}{5}\right)$$

$$= \ln 5 + \ln\left(1-\left(\frac{5-x}{5}\right)\right)$$

$$= \ln 5 - \sum_{n=1}^{\infty} \left(\frac{5-x}{5}\right)^n \Big/ n$$

$$= \ln 5 - \sum_{n=1}^{\infty} \frac{(-1)^n}{5^n \cdot n}(x-5)^n$$