Q: $\sum_{n=1}^{\infty}\left(\frac{6n^6}{5n^2+8}\right)^p$ converges <u>for which</u> values of p?

A: Idea: for large n, $5n^2 \gg 8$

so $5n^2+8 \approx 5n^2$ (e.g. if $n=1000$, $5n^2 = 5000000$, $5n^2+8 = 5000008$)

$$\text{so } \underbrace{\left(\frac{6n^6}{5n^2+8}\right)^p}_{a_n} \approx \left(\frac{6n^6}{5n^2}\right)^p = \underbrace{\left(\frac{6}{5}n^4\right)^p}_{b_n}$$

so use Limit-Comparison to reduce from $\sum a_n$ to $\sum b_n$.

(to check that this is allowed,

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\left(\frac{6n^6}{5n^2+8}\right)^p}{\left(\frac{6n^6}{5n^2}\right)^p} = \lim_{n\to\infty}\left(\frac{5n^2}{5n^2+8}\right)^p = 1$$
)

$$\text{Now, } \sum b_n = \sum \left(\frac{6}{5}n^4\right)^p = \sum \left(\frac{6}{5}\right)^p n^{4p}$$

$$= \left(\frac{6}{5}\right)^p \sum n^{4p}$$

$$= \left(\frac{6}{5}\right)^p \sum \frac{1}{n^{-4p}}$$

and by p-test, this $\begin{cases} \underline{\text{converges}} \text{ if } -4p > 1 \\ \underline{\text{diverges}} \text{ if } -4p \leq 1 \end{cases}$

<u>converges</u> if $-4p > 1$

i.e. if $\underline{\underline{p < -\frac{1}{4}}}$

(NB: multiplying both sides by $-\frac{1}{4}$ reverses the $>$ to $<$)