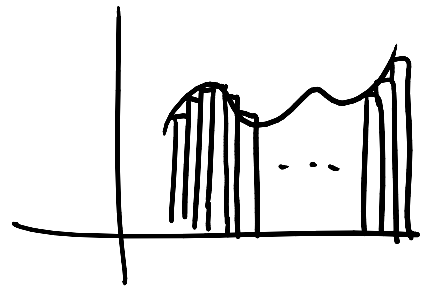
Lecture 4 — 27 Jan 2010

Housekeeping: HW01 due 1/29 3am (this Fri morning)
HW03 due 2/2 3am (next Tue morning)

Last time: computing areas under curves $y=f(x)$ by approximating the region as a union of n rectangles and then taking $n\to\infty$.

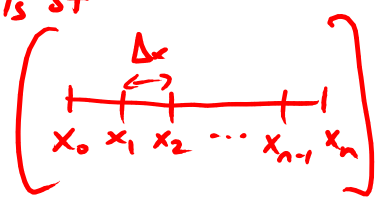


Got $A = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\, f(x_i^*)$

x_i^* means the sample point in the i-th interval: could be the left, right, midpoint, etc.

Definite integrals (Ch 5.2)

Definition. Say $f(x)$ is a function defined for $a \le x \le b$.
Divide the interval $[a,b]$ into n equal subintervals of equal width Δx, with endpoints $x_0, x_1, \ldots, x_n$, where $x_0 = a$ and $x_n = b$.



Pick any "sample points" x_i^* in $[x_{i-1}, x_i]$

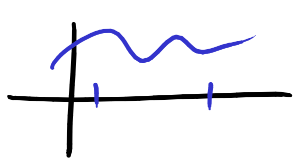
The definite integral of f from a to b is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

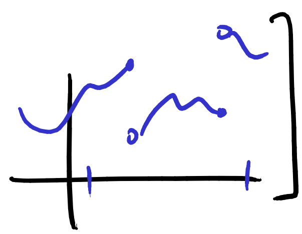
"Riemann sums"

if that limit exists! (If it does, we call f integrable on $[a,b]$.)

[Most f that we encounter in real life are integrable:
e.g. f continuous



or even f with finite # of jumps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Example: Write the definition of $\int_1^3 \sqrt{x}\, dx$.

Divide the interval $[1,3]$ into n equal subintervals with width $\Delta x = \frac{3-1}{n} = \frac{2}{n}$

endpoints $x_0 = 1,\ x_1 = 1+\Delta x,\ x_2 = 1+2\Delta x, \dots$

$x_i = 1 + i\Delta x = 1 + \frac{2i}{n}$

$$\int_1^3 \sqrt{x}\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\, \Delta x$$

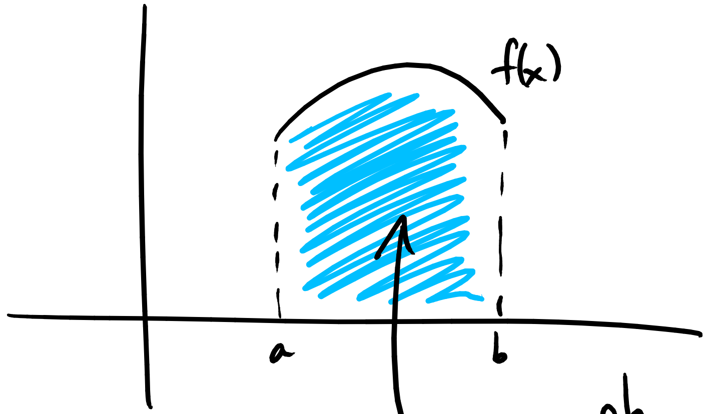
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1+\frac{2i}{n}}\ \frac{2}{n}$$

$$\left[= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \sqrt{1+\frac{2i}{n}} \right.$$

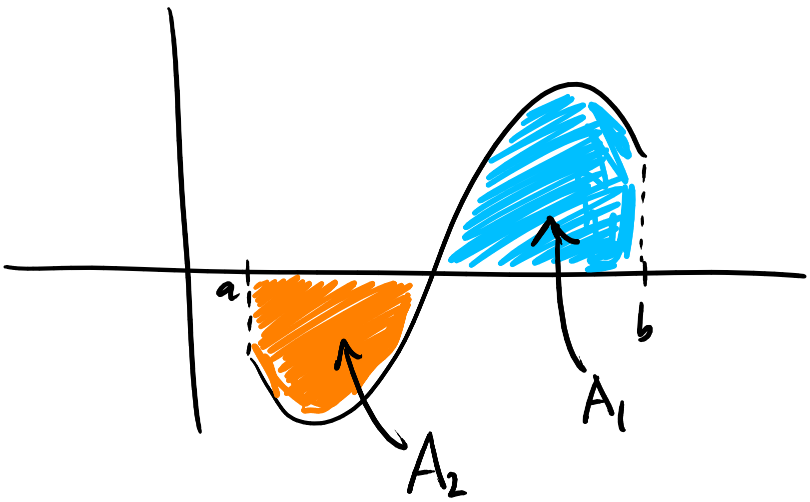
by the general rule

$$\left. \sum_{i=1}^{n} c\cdot a_i = c \sum_{i=1}^{n} a_i \right]$$

Integrals compute areas:



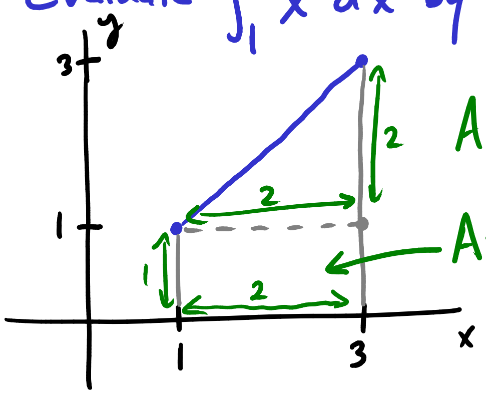
$$\int_a^b f(x)\,dx = A$$



$$\int_a^b f(x)\,dx = A_1 - A_2$$

$$\int_a^b |f(x)|\,dx = A_1 + A_2$$

Example. Evaluate $\int_1^3 x\,dx$ by interpreting it in terms of areas.



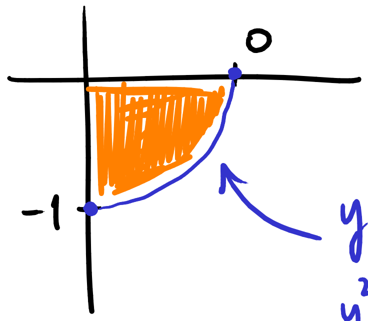
$A = \frac{1}{2}(2\times 2) = 2$

$A = 2\cdot 1 = 2$

total area $2+2=4$

so $\underline{\underline{\int_1^3 x\,dx = 4}}$

Example. Evaluate $\int_0^1 -\sqrt{1-x^2}\,dx$ by interp. it in terms of areas.



$y = -\sqrt{1-x^2}$

$y^2 = 1 - x^2$

$y^2 + x^2 = 1 \longrightarrow$ circle of radius 1

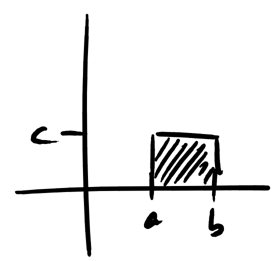
$$\text{area} = \frac{1}{4}(\text{area of circle of radius } 1) = \frac{1}{4}(\pi \cdot 1^2) = \frac{\pi}{4}$$

So $\int_0^1 -\sqrt{1-x^2}\,dx = -\frac{\pi}{4}$

minus sign b/c the function is negative on $[0,1]$

A few basic facts about integrals:

1) $\int_a^b c\,dx = c(b-a)$

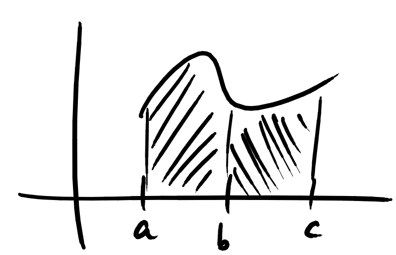


2) $\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3) $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

4) $\int_a^b f(x)-g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

5) $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



Definition. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$.