Lecture 11

12 Feb 2010

1^{st} midterm Feb 23 (week from Tue) 7–9pm

Office hours M 1:30–2:30p

F 10:00–11:00a ← CHANGED RLM 9.134

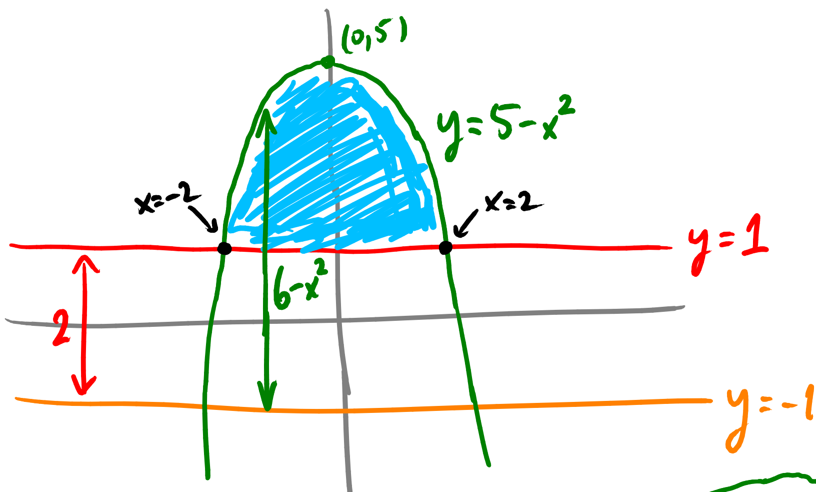
Lecture next M covered by Prof. Daniel Allcock

Ex: Find the volume of solid obtained by rotating the region between $y = 5 - x^2$ and $y = 1$ around the line $y = -1$.

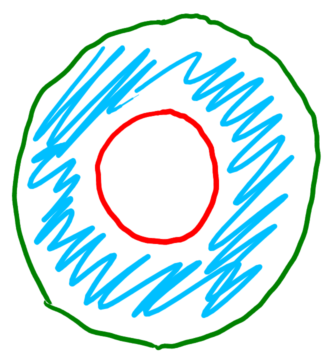
Intersection pts:

$1 = 5 - x^2$

$x = \pm 2$



Cross sections at fixed x look like washers:



radius of inner circle = 2

" " outer circle = $6 - x^2$

$$V = \int_{-2}^{2} A(x)\,dx = \int_{-2}^{2} \pi\left((6-x^2)^2\right) - \pi\left(2^2\right)\,dx$$

$$= \frac{384\pi}{5}$$

Limits of integration and u-substitution

Ex Find $\int_1^e \frac{\ln x}{x}\,dx$.

Take $u = \ln x$: $du = \frac{dx}{x}$

$x\,du = dx$

$x=1 \Rightarrow u = \ln 1 = 0$

$x=e \Rightarrow u = \ln e = 1$

$$\int_0^1 u\,du = \frac{1}{2}u^2\Big|_0^1 = \frac{1}{2}$$

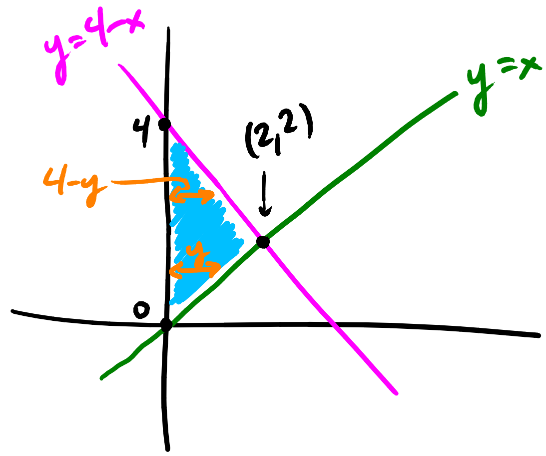
Ex Find the volume of the solid obtained by rotating the region between

$y=x$

$y=4-x$

and the y-axis

around the y-axis.



$y=4-x$

$x=4-y$

$$V = \int_0^4 dy\, A(y) = \int_0^2 dy\, A(y) + \int_2^4 dy\, A(y)$$

$$= \int_0^2 dy\, \pi y^2 + \int_2^4 dy\, \pi(4-y)^2$$

$$= \frac{8\pi}{3} + \frac{8\pi}{3}$$

$$= \frac{16\pi}{3}$$

Ex Find $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,dx$.

Try $u = 1+\cos^2 x$.

$$\frac{du}{dx} = (-\sin x)(2\cos x) \rightarrow du = -2\sin x\cos x\,dx$$
$$= -\sin 2x\,dx$$

So $$\int \frac{\sin 2x}{1+\cos^2 x}\,dx = \int \frac{-du}{u} = -\int \frac{1}{u}\,du$$
$$= -\ln|u| + C$$
$$= -\ln|1+\cos^2 x| + C$$
$$= -\ln(1+\cos^2 x) + C \quad \left(= \ln \frac{1}{1+\cos^2 x}\right)$$

$$-\ln A = \ln \frac{1}{A}$$

Ex $$\int_0^3 \frac{5x^2+10x+2}{10x^2+4}\,dx$$

Might first try $u = 10x^2+4$

then $du = 20x\,dx$ ie $dx = \dfrac{du}{20x}$

$$\int_4^{94} \frac{5x^2+10x+2}{u}\,\frac{du}{20x}$$

Looks hard — try splitting the integral up:

$$\int_0^3 \frac{5x^2+2}{10x^2+4}\,dx + \int_0^3 \frac{10x}{10x^2+4}\,dx$$

$$= \int_0^3 \frac{5x^2+2}{2(5x^2+2)}\,dx + \int_0^3 \frac{10x}{10x^2+4}\,dx$$

$$= \int_0^3 \frac{1}{2}\,dx + \int_0^3 \frac{10x}{10x^2+4}\,dx$$

easy $\left(=\frac{3}{2}\right)$

u-subst. $u = 10x^2+4$

gives $\frac{1}{2}\ln\left(\frac{47}{2}\right)$

Ex

$$\int e^x(4+e^x)^3\,dx$$

could just multiply it out

(painful!)

But substitute $u = 4+e^x$

$du = e^x\,dx$

$dx = \frac{du}{e^x}$

$$\rightarrow \int e^x u^3 \frac{du}{e^x} = \int u^3\,du = \frac{u^4}{4} + C = \frac{(4+e^x)^4}{4} + C$$