Tests for Series

Does $\sum a_n$ converge?

[$\sum$ always means $\sum_{n=M}^{\infty}$ below]

1) "Test for Divergence": If $\lim_{n\to\infty} a_n$ doesn't exist or $\lim_{n\to\infty} a_n \neq 0$, Then $\sum a_n$ diverges

2) Geometric Series Test: If $\sum a_n$ is a geometric series with common ratio r, Then $\sum a_n$ $\begin{cases} \text{converges} & \text{if } |r|<1 \\ \text{diverges} & \text{if } |r| \geq 1 \end{cases}$

3) Integral Test. If $f(x)$ is a decreasing positive function and $a_n = f(n)$, Then $\sum a_n$ and $\int_M^\infty f(x)\,dx$ either both converge or both diverge.

4) p-test. If $a_n = \frac{1}{n^p}$, Then $\sum a_n$ $\begin{cases} \text{converges} & \text{if } p>1 \\ \text{diverges} & \text{if } p \leq 1 \end{cases}$

5) Comparison Tests.

a) If $a_n, b_n \geq 0$, $a_n \geq b_n$, and $\sum b_n$ diverges, Then $\sum a_n$ diverges.

b) If $a_n, b_n \geq 0$, $a_n \leq b_n$, and $\sum b_n$ converges, Then $\sum a_n$ converges.

6) Limit Comparison Test. If $a_n, b_n \geq 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ with $c \neq 0$, Then $\sum a_n$ and $\sum b_n$ either both converge or both diverge.

7) Alternating Series Test.

If $\lim_{n\to\infty} b_n = 0$ and b_n are positive, decreasing ($b_{n+1} \le b_n$)

Then $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.

8) Ratio Test.

a) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$ Then $\sum a_n$ converges absolutely.

b) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $= \infty$ Then $\sum a_n$ diverges.

9) Root Test.

a) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$ Then $\sum a_n$ converges absolutely.

b) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or $= \infty$ Then $\sum a_n$ diverges.