# Lecture 43

Final: 3 hours (7-10pm) next Thursday

25 problems (8 on seq/series)
(3 on multivariate)

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## Final Review

- If $f''(x) = \sqrt{x}$, $f(0) = 1$, $f'(0) = 2$

  What is $f(1)$?

$$f'(x) = \frac{2}{3} x^{3/2} + C$$

$$f(x) = \frac{4}{15} x^{5/2} + Cx + D$$

To get C and D: $f(0) = \frac{4}{5}(0)^{5/2} + C(0) + D = 1$

so $D = 1$

$f'(0) = \frac{2}{3}(0)^{3/2} + C = 2$

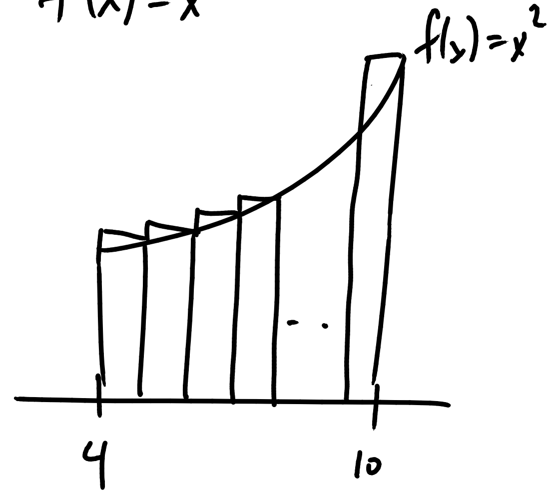
so $C = 2$

So $f(x) = \frac{4}{15} x^{5/2} + 2x + 1$

$f(1) = \frac{4}{15} + 2 + 1 = \underline{\underline{\frac{49}{15}}}$

- Write a formula for the area under the curve $f(x)=x^2$ on the interval $[4,10]$.

(using right endpoints)

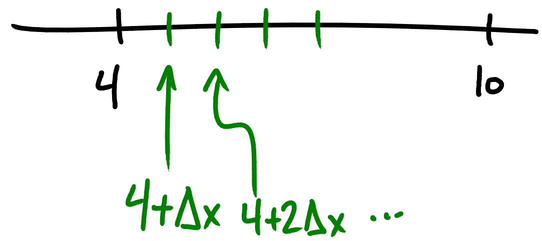



Chop $[4,10]$ into $n$ intervals:

width $\Delta x = \frac{10-4}{n} = \frac{6}{n}$

right endpt of $i^{th}$ interval $4+i\Delta x = 4+\frac{6i}{n} = x_i$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$




$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4+\frac{6i}{n}\right)^2 \frac{6}{n}$$

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- If $g(x) = \int_{30}^{x^2} \tan u \, du$

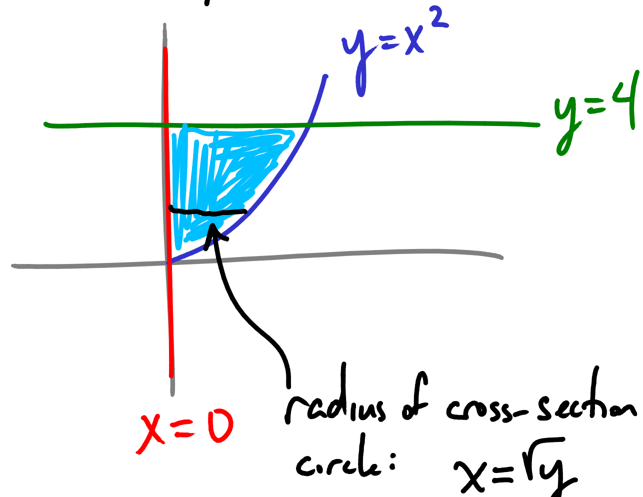
what is $g'(x)$?

Use FTC: $\frac{d}{dx}\int_{30}^{x} f(u)\,du = f(x)$

and remember chain rule: $\frac{d}{dx} g(x) = 2x \tan x^2$

• Find the volume of the solid obtained by rotating the region bounded by

$y=x^2$
$x=0$
$y=4$




around the y-axis.

Chop the solid into cross-sections at fixed $y$.

Each x-section is a circle with radius $=\sqrt{y}$

So $V=\int_0^4 \pi r^2\,dy=\int_0^4 \pi y\,dy=\pi\frac{y^2}{2}\Big|_0^4=\underline{\underline{8\pi}}$

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Methods of integration:

Remember $\int$ by parts

• Find $\int 6x(\ln x)^2\,dx$

IBP: $u=(\ln x)^2 \quad v=3x^2$

$du=2\frac{\ln x}{x}dx \quad dv=6x\,dx$

$uv-\int v\,du=(\ln x)^2 3x^2-\int 3x^2\cdot 2\frac{\ln x}{x}\,dx$

$$= (\ln x)^2 3x^2 - \int 6x \ln x \, dx$$

IBP again: $u = \ln x \quad v = 3x^2$

$du = \frac{1}{x} dx \quad dv = 6x \, dx$

$$= (\ln x)^2 3x^2 - \left(uv - \int v \, du\right)$$

$$= (\ln x)^2 3x^2 - 3x^2 \ln x + \int 3x^2 \frac{dx}{x}$$

$$= (\ln x)^2 3x^2 - 3x^2 \ln x + \frac{3}{2}x^2 + C$$

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Find $\displaystyle\int_0^{\pi/3} \frac{\sec x \tan x}{5 - \sec x} dx$

Remember

$d(\sec x) = \sec x \tan x \, dx$

Substitute $u = 5 - \sec x$

$du = -\sec x \tan x \, dx$

$x = 0$ is $u = 5 - 1 = 4$

$x = \pi/3$ is $u = 5 - 2 = 3$

$$\int_4^3 \frac{-du}{u} = -\ln(u) \Big|_{u=4}^{u=3}$$

$$= -\left[\ln(3) - \ln(4)\right] = -\ln\left(\frac{3}{4}\right) = \ln\left(\frac{4}{3}\right)$$

Remember basic facts about integrals:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

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Partial fractions can always be used for $\int \frac{P(x)}{Q(x)}\,dx$

[ex $\int \frac{2x^2+3x+1}{x^2+x-2}\,dx$ ]

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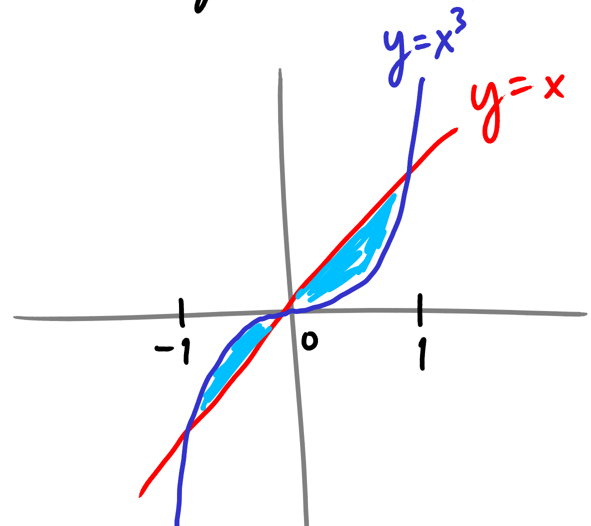
Area between curves:

What's the area enclosed by the graphs of $y=x$ and $y=x^3$?

Solve $x=x^3$ to find the intersections:

$$x^3-x=0$$
$$x(x^2-1)=0$$
$$x(x+1)(x-1)=0$$




$$A=\int_{-1}^{0}(x^3-x)\,dx + \int_0^1 (x-x^3)\,dx$$

$$= \frac{1}{4} + \frac{1}{4} = \underline{\underline{\frac{1}{2}}}$$

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## Improper integrals

Just remember $\int_a^\infty f(x)\,dx$ **means** $\lim\limits_{t\to\infty} \int_a^t f(x)\,dx$

**Ex** $$\int_2^\infty \sin(x)\,dx = \lim_{t\to\infty} \int_2^t \sin(x)\,dx$$

$$= \lim_{t\to\infty} -\cos(x)\Big|_2^t$$

$$= \lim_{t\to\infty} \big(-\cos(t) + \cos(2)\big)$$

<u>doesn't exist</u> (so $\int$ diverges)

**Ex** $$\int_0^\infty \frac{1}{1+x^2}\,dx = \lim_{t\to\infty} \int_0^t \frac{1}{1+x^2}\,dx = \lim_{t\to\infty} \tan^{-1}(t) - \tan^{-1}(0)$$

$$= \frac{\pi}{2} - 0 = \underline{\underline{\frac{\pi}{2}}}$$