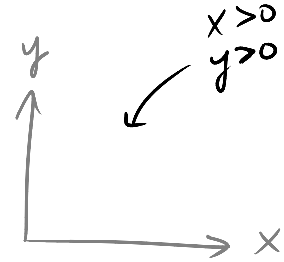
Lecture 4

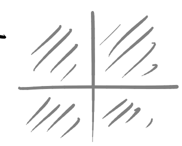
9 Sep 2014

Coordinates in 3 dimensions (Ch 12.1)

Up to now, we worked in the plane

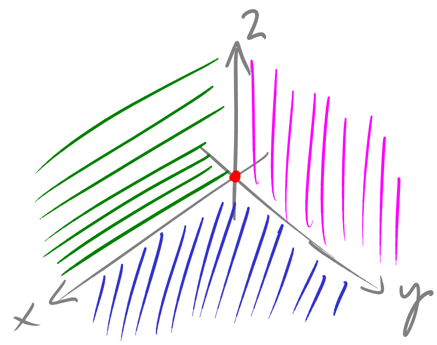


(divided into quadrants


)

Now, work in 3 dimensions:

space is divided into 8 regions ("octants")



positive octant: $x > 0$
$y > 0$
$z > 0$

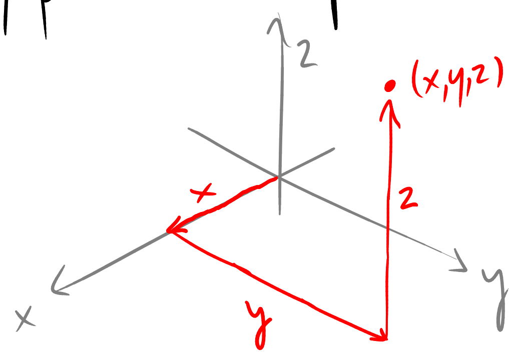
like a room, origin $(x,y,z) = (0,0,0)$ at corner

x-y plane is floor: $z = 0$

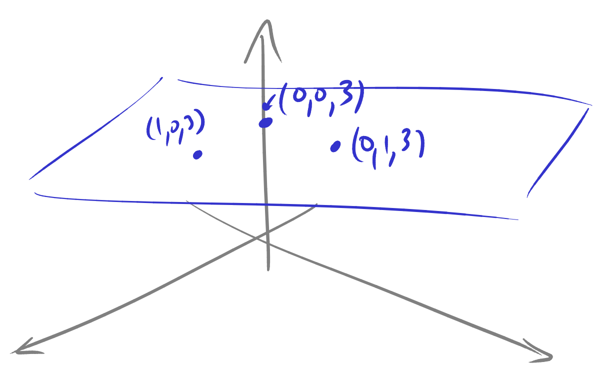
x-z plane is a wall: $y = 0$

y-z plane is a wall: $x = 0$

Any point in 3-dim space can be specified by 3 #'s (x,y,z)



Ex Draw the locus $\{z=3\}$ in 3 dim.
(i.e. the set of all points obeying this equation)

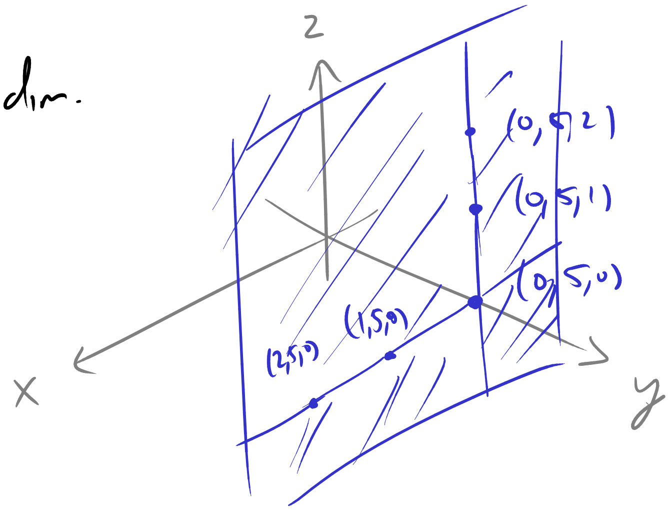


It's a plane.

Rk Imposing 1 equation on (x,y,z) usually gives something 2-dimensional.

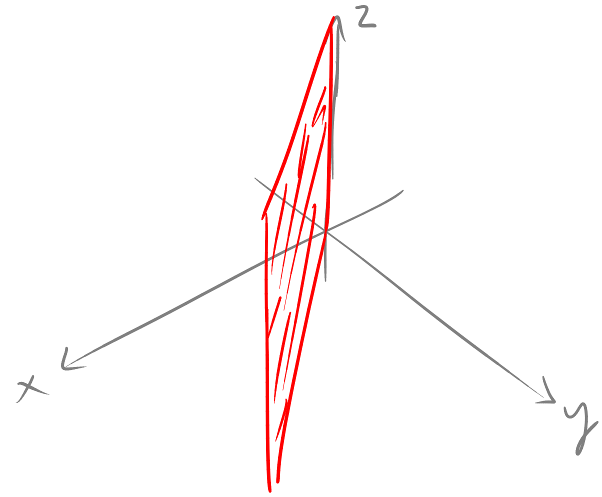
Ex Draw the locus $\{y=5\}$ in 3 dim.

Again a plane.



Ex Draw $\{x=y\}$ in 3 dim.

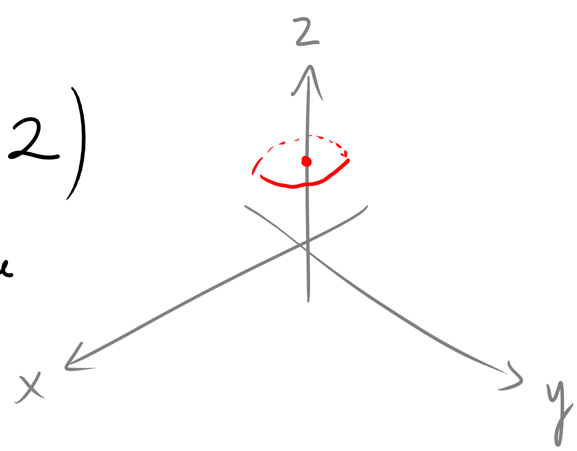
Get a plane lying over the line $x=y$ in the xy-plane.



[In fact any eq. of the form $Ax+By+Cz+D=0$ gives a plane.]

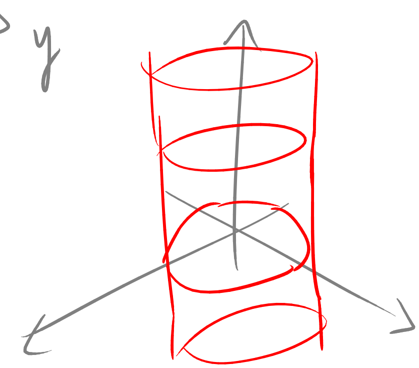
Ex $(x^2+y^2=1$ and $z=2)$

→ circle in the $z=2$ plane
center at $(0,0,2)$
radius 1



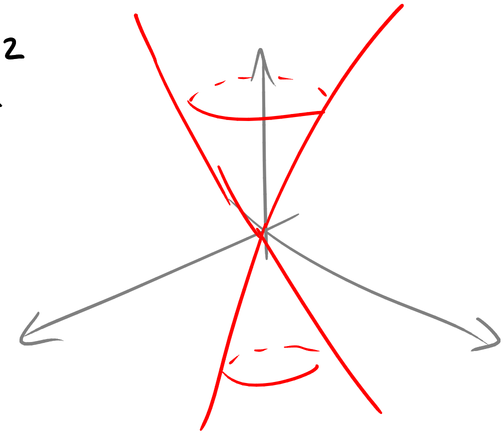
Ex $x^2+y^2=9$

→ cylinder around z-axis, radius 3



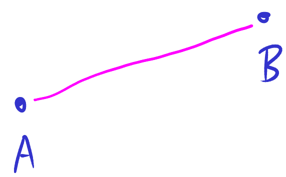
Ex $x^2+y^2=z^2$

"double cone"



Ex $z=3$: $x^2+y^2=3^2$
circle radius 3

Distance Given two points $A=(x_1,y_1,z_1)$ $B=(x_2,y_2,z_2)$



the distance from A to B is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$

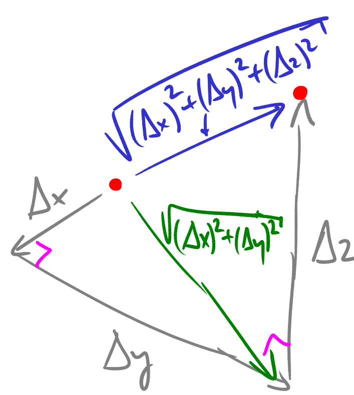
$$= \sqrt{(\Delta x)^2+(\Delta y)^2+(\Delta z)^2}$$

Ex Distance from $(1,-1,4)$ to $(2,3,0)$ is

$$\sqrt{(1-2)^2+(-1-3)^2+(4-0)^2} = \sqrt{1+16+16} = \sqrt{33}$$

Why does this formula work?

Just Pythag. Thm twice!



Ex What is the locus $\{x^2+y^2+z^2=25\}$?

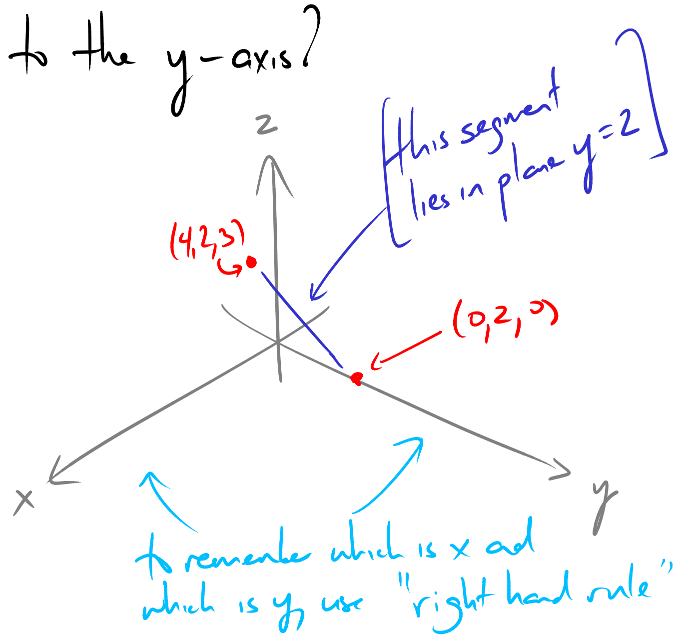
$x^2+y^2+z^2$ is the distance from $(0,0,0)$ to (x,y,z). So,

we have a sphere centered at $(0,0,0)$ with radius $= 5$.

Similarly: $(x-h)^2+(y-k)^2+(z-l)^2 = r^2$ is sphere

centered at (h,k,l) w/ radius r.

Ex What is the distance from $(4,2,3)$ to the y-axis?



The y-axis is given by

$x=0,\ z=0$.

So, any point on the y-axis

is $(0,y,0)$.

Dist. from $(4,2,3)$ to $(0,y,0)$ is $\sqrt{(4-0)^2+(2-y)^2+(3-0)^2}$

$$= \sqrt{25+(2-y)^2}$$

The minimum distance is attained by setting $y=2$.

Then, set distance $= \sqrt{25} = 5$.

Vectors (Ch 12.2)

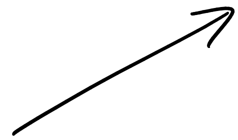
A vector is a quantity with magnitude and direction.

Picture it as an arrow:

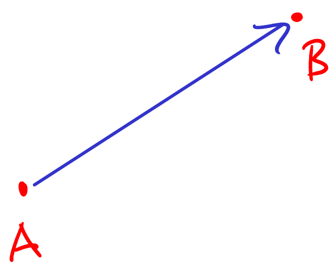
length and direction

are the only information

it knows (doesn't care where it starts)



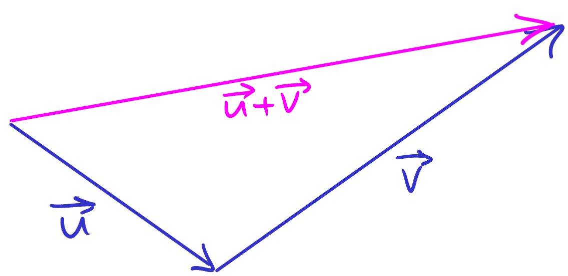
Ex If have two points A, B:



"displacement vector"

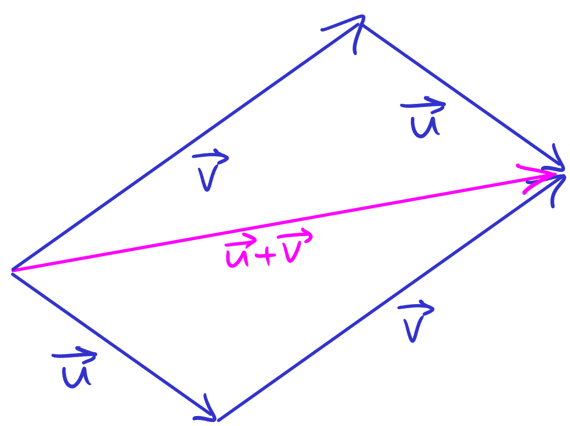
from A to B $\overrightarrow{AB}$

Adding vectors Given vectors $\vec{u}, \vec{v}$ define a new vector $\vec{u}+\vec{v}$



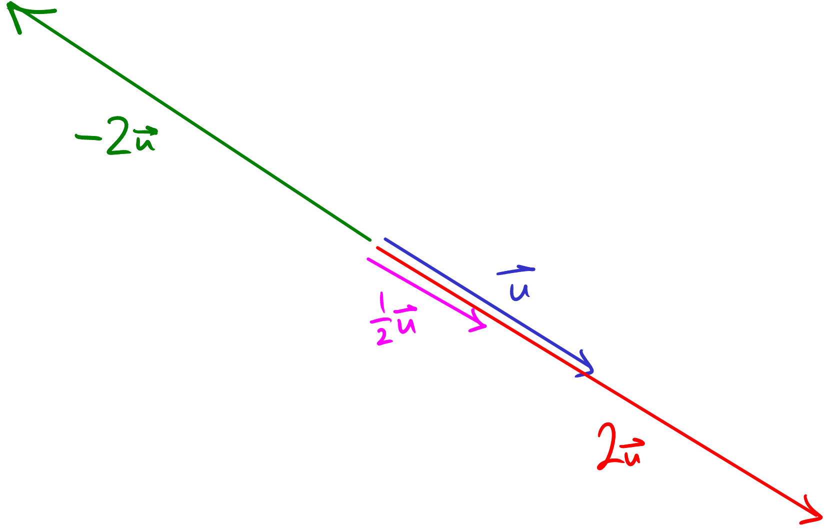
(also called "parallelogram law":

$\vec{u}+\vec{v} = \vec{v}+\vec{u}$)



Scalar multiplication: Given a vector $\vec{u}$ and a real number λ,

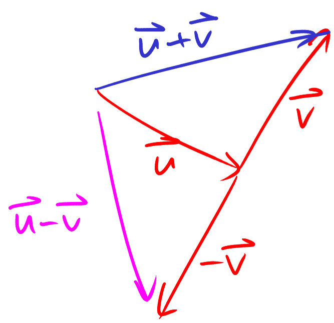
define $\lambda\vec{u}$ to be a vector which: if $\lambda > 0$, points same direction as $\vec{u}$ and has length λ times the length



of $\vec{u}$.

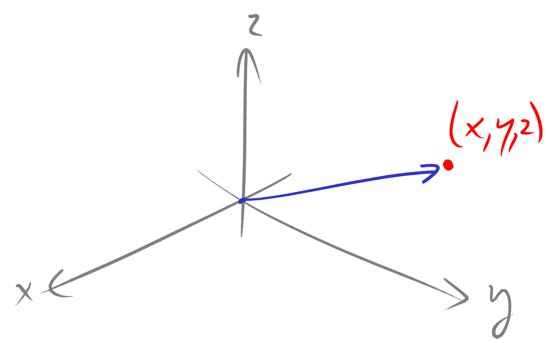
if $\lambda < 0$, points opposite dir from $\vec{u}$, length $|\lambda|$ times the length of $\vec{u}$.

Subtraction $\quad \vec{u} - \vec{v} = \vec{u} + (-1 \cdot \vec{v}) \qquad -\vec{v} = (-1) \cdot \vec{v}$



How to represent a vector concretely? $\quad \vec{v}$

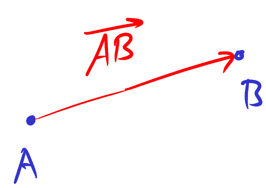
Put base of $\vec{v}$ at $(0,0,0)$ in (x,y,z) coords.



Call the coords of the point at the tip of $\vec{v}$ the "components" of $\vec{v}$. Write $\vec{v} = \langle x, y, z \rangle$

Ex Say $A = (1,2,-4) \quad B = (3,4,7)$

What is $\overrightarrow{AB}$? $\quad \langle 2, 2, 11 \rangle$



Ex Say $A=(1,3,8)$ $B=(0,5,6)$

What is $\overrightarrow{AB}$? $\langle -1,2,-2\rangle$ $\left[\overrightarrow{BA}=\langle 1,-2,2\rangle\right]$

Generally if $A=(x_1,y_1,z_1)$, $B=(x_2,y_2,z_2)$ then $\overrightarrow{AB}=\langle x_2-x_1, y_2-y_1, z_2-z_1\rangle$