Corrections: *The local convexity of solving systems of quadratic equations*

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This note corrects a few errors from proofs in *The local convexity of solving systems of quadratic equations*. The results do not change, except for small modifications to some values of constants. We refer to the latest version on the arxiv, dated June 2, 2016.

- The statement of Theorem 2.1 (Strong Convexity) is correct, except that the constant of $\frac{3}{10}$ in (8) should be replaced by $\frac{1}{10}$, and the constant of $\frac{1}{18}$ in the subsequent inequality should be replaced by $\frac{3}{100}$. In the proof (starting on p. 23), the random variable A_i is not a chi-square random variable with 1 degree of freedom, but rather a *convex combination* of r independent chi-square random variables, each with 1 degree of freedom. One easily calculates that $\mathbb{E}[A_i] = 1$, and that $1 \leq \mathbb{E}[A_i^2] \leq 3$. Moreover, the higher moments $\mathbb{E}[A_i^4], \mathbb{E}[A_i^6], \mathbb{E}[A_i^8]$, and $\mathbb{E}[A_i^1 2]$ are all bounded by absolute constants. These properties suffice for the proof to carry through, up to a small change in the resulting constants stemming from using the lower bound $1 \leq \mathbb{E}[A_i^2]$.

- The statement of Lemma 3.11 (Initialization) is correct, but part of the proof is incorrect. One way to correctly bound $d(U_0)$ in terms of δ is as follows: Using Lemma 5.4 in [1] and then Weyl's matrix inequality, we have under the stated number of measurements m that

$$\begin{aligned} d(U_0) &\leq \frac{1}{2(\sqrt{2}-1)\lambda_r} \|U_0 U_0^T - XX^T\|_F^2 \\ &\leq \frac{r}{2(\sqrt{2}-1)\lambda_r} \|U_0 U_0^T - XX^T\|_{op}^2 \\ &\leq \frac{r}{(\sqrt{2}-1)\lambda_r} \left(\|A - XX^T\|_{op}^2 + |\sigma_{r+1} - 1/2|^2 \right) \\ &\leq \frac{r\delta^2}{2(\sqrt{2}-1)\lambda_r}; \end{aligned}$$

  the remainder of the proof proceeds correctly.

References

[1] Stephen Tu, Ross Boczar, Max Simchowitz, Mahdi Soltanolkotabi, and Ben Recht. Low-rank Solutions of Linear Matrix Equations via Procrustes Flow. *International Conference on Machine Learning*, 2016: 964–973.