FIVE MINUTE REVIEW FOR WEEK 3.

Question 1. *Simplify $e^{9\log(\pi)}$.*

Answer 1.
$$e^{9\log(\pi)} = e^{\log(\pi^9)} = \pi^9$$

Question 2. *Simplify $1 - \frac{1}{6} - \frac{1}{7}$.*

Answer 2.
$$1 - \frac{1}{6} - \frac{1}{7} = \frac{42 - 7 - 6}{42} = \frac{42 - 13}{42} = \frac{29}{42}$$

Question 3. *Simplify $e^{4\log|x|} - (x^2+1)(x^2-1)$.*

Answer 3.
$$e^{4\log|x|} - (x^2+1)(x^2-1) = e^{\log(x^4)} - (x^4-1) = x^4 - x^4 + 1 = 1$$

Question 4. *Simplify $\frac{\sin(2\theta)}{2\sin(\theta)}$.*

Answer 4.
$$\frac{\sin(2\theta)}{2\sin(\theta)} = \frac{2\sin\theta\cos\theta}{2\sin\theta} = \cos\theta$$

Question 5. $\frac{d}{dx}\left((x-2)^{3/2}\right) = ?$

Answer 5.
$$\frac{d}{dx}\left((x-2)^{3/2}\right) = \frac{3}{2}(x-2)^{\frac{1}{2}}$$

Question 6. $\frac{d}{dt}\int_3^t 2^s ds = ?$

Answer 6.
$$\frac{d}{dt}\int_3^t 2^s ds = 2^t$$

Question 7. *State the product rule.*

Answer 7.
$$\frac{d}{dx}\left(f(x)g(x)\right) = \frac{df}{dx}g(x) + f(x)\frac{dg}{dx}$$

Question 8. $\frac{d}{d\theta}\int_0^{\theta^2}\cos\phi\, d\phi = ?$

Answer 8. Let $u = \theta^2$. Then,
$$\frac{d}{d\theta}\int_0^{\theta^2}\cos\phi\, d\phi = \frac{du}{d\theta}\frac{d}{du}\int_0^u \cos\phi\, d\phi = 2\theta\cos(u) = 2\theta\cos(\theta^2)$$

Question 9. *State the Fundamental Theorem of Calculus.*

Answer 9. (a) Let f be integrable on $[a,b]$, and define a function F on $[a,b]$ by
$$F(x) = \int_a^x f(t)dt.$$

If f is continuous at $c \in [a,b]$, then F is differentiable at c, and
$$\frac{dF}{dx}(c) = f(c).$$

Date: September 8, 2015.

(b) If f is integrable on $[a, b]$ and $f(t) = \frac{dg}{dt}$ for some function $g(t)$, then

$$\int_a^b f(t)dt = g(b) - g(a).$$

Question 10. *State the chain rule.*

Answer 10.
$$\frac{d}{dx}F(u(x)) = \frac{dF}{du}\frac{du}{dx}$$

Question 11. $\int_1^2 (t^5 - t^7)dt = ?$

Answer 11.
$$\int_1^2 (t^5 - t^7)dt = \left[\frac{t^6}{6} - \frac{t^8}{8}\right]_1^2 = \frac{2^6}{6} - \frac{2^8}{8} - \frac{1}{6} + \frac{1}{8} = 2^5\left(\frac{1}{3} - 1\right) + \frac{3 - 4}{24} = -\frac{2^6}{3} - \frac{1}{24} = -\left(\frac{2^9 + 1}{24}\right)$$

Question 12. $\int_1^e \left(\frac{1}{t} - t\right) dt = ?$

Answer 12.
$$\int_1^e \left(\frac{1}{t} - t\right) dt = \left[\log|t| - \frac{t^2}{2}\right]_1^e = 1 - \frac{e^2}{2} + \frac{1}{2} = \frac{3 - e^2}{2}$$