FIVE MINUTE REVIEW FOR WEEK 4.

Question 1. *Simplify $e^t e^{-t}$.*

Answer 1.
$$e^t e^{-t} = e^0 = 1$$

Question 2. *Simplify $\frac{1+\frac{1}{c-1}}{1-\frac{1}{c-1}}$.*

Answer 2.
$$\frac{1+\frac{1}{c-1}}{1-\frac{1}{c-1}} = \left(\frac{c-1}{c-1}\right) \cdot \left(\frac{1+\frac{1}{c-1}}{1-\frac{1}{c-1}}\right) = \frac{c-1+1}{c-1-1} = \frac{c}{c-2}$$

Question 3. *Expand and simplify $(1 + x - x^2)^2$.*

Answer 3.
$$(1 + x - x^2)^2 = 1 + x - x^2 + x + x^2 - x^3 - x^2 - x^3 + x^4 = 1 + 2x - x^2 - 2x^3 + x^4$$

Question 4. *State the Fundamental Theorem of Calculus.*

Answer 4. (a) Let f be integrable on $[a, b]$, and define a function F on $[a, b]$ by
$$F(x) = \int_a^x f(t)dt.$$

If f is continuous at $c \in [a, b]$, then F is differentiable at c, and
$$\frac{dF}{dx}(c) = f(c).$$

(b) If f is integrable on $[a, b]$ and $f(t) = \frac{dg}{dt}$ for some function $g(t)$, then
$$\int_a^b f(t)dt = g(b) - g(a).$$

Question 5. $\frac{d}{dt}(\sin(e^t)) = ?$

Answer 5.
$$\frac{d}{dt}(\sin(e^t)) = e^t \cos(e^t)$$

Question 6. $\frac{d}{dx}\left(e^{7x}x^{-7}\right) = ?$

Answer 6.
$$\frac{d}{dx}\left(e^{7x}x^{-7}\right) = \frac{d}{dx}(e^{7x})x^{-7} + e^{7x}\frac{d}{dx}(x^{-7}) = 7e^{7x}x^{-7} - 7e^{7x}x^{-8} = \frac{7e^{7x}}{x^8}(x-1)$$

Question 7. $\int_0^\varphi e^\theta(\sin\theta + \cos\theta)d\theta = ?$

Answer 7.
$$\int_0^\varphi e^\theta(\sin\theta + \cos\theta)d\theta = \int_0^\varphi \frac{d}{d\theta}(e^\theta \sin\theta)d\theta = e^\varphi \sin\varphi - e^0 \sin(0) = e^\varphi \sin\varphi$$

Question 8. $\frac{d}{dy}\int_{\sqrt{\pi}}^y \frac{\chi^{70}}{4!}d\chi = ?$

Answer 8.
$$\frac{d}{dy}\int_{\sqrt{\pi}}^y \frac{\chi^{70}}{4!}d\chi = \frac{y^{70}}{4!}$$

Question 9. $\int_0^v \frac{du}{4+u^2} = ?$

Answer 9. Let $u = 2\tan\theta$ so that $4 + u^2 = \frac{4}{\cos^2\theta}$ and $du = \frac{2d\theta}{\cos^2\theta}$. Then

$$\int_0^v \frac{du}{4+u^2} = \int_0^{\arctan\left(\frac{v}{2}\right)} \frac{\cos^2\theta}{4}\frac{2d\theta}{\cos^2\theta} = \frac{1}{2}\int_0^{\arctan\left(\frac{v}{2}\right)} d\theta = \frac{1}{2}\arctan\left(\frac{v}{2}\right)$$

Question 10. $\frac{d}{da}\int_1^{e^a} \log(\xi)d\xi = ?$

Answer 10. Let $b = e^a$. Then

$$\frac{d}{da}\int_1^{e^a} \log(\xi)d\xi = \frac{db}{da}\frac{d}{db}\int_1^b \log(\xi)d\xi = e^a\log(b) = e^a\log(e^a) = ae^a$$