FIVE MINUTE REVIEW FOR WEEK 5.

Question 1. *Simplify* $1 + \frac{1}{n+1} + \frac{1}{n-1}$.

Answer 1.

$$1 + \frac{1}{n+1} + \frac{1}{n-1} = 1 + \frac{n-1+n+1}{n^2-1} = \frac{n^2+2n-1}{n^2-1}$$

Question 2. *Simplify* $\frac{\sqrt{a}}{a^5} + a^{-\frac{9}{2}} - \frac{3}{2\sqrt{a}[(a^2+1)(a^2-1)+1]}$.

Answer 2.

$$\frac{\sqrt{a}}{a^5} + a^{-\frac{9}{2}} - \frac{3}{2\sqrt{a}[(a^2+1)(a^2-1)+1]} = 2a^{-9/2} - \frac{3}{2\sqrt{a}(a^4-1+1)} = 2a^{-9/2} - \frac{3}{2}a^{-9/2} = \frac{1}{2}a^{-9/2}$$

Question 3. *Solve* $x^2 - \sqrt{\frac{2}{3}}\left(1+\sqrt{\frac{2}{3}}\right)x + \sqrt{\frac{8}{27}}$.

Answer 3. Use the quadratic formula:

$$\begin{aligned} x &= \frac{\sqrt{\frac{2}{3}}\left(1+\sqrt{\frac{2}{3}}\right) \pm \sqrt{\frac{2}{3}\left(1+\sqrt{\frac{2}{3}}\right)^2 - 4\sqrt{\frac{8}{27}}}}{2} \\ &= \frac{1}{2}\sqrt{\frac{2}{3}}\left(1+\sqrt{\frac{2}{3}} \pm \sqrt{\left(1+\sqrt{\frac{2}{3}}\right)^2 - 4\sqrt{\frac{2}{3}}}\right) \\ &= \frac{1}{2}\sqrt{\frac{2}{3}}\left(1+\sqrt{\frac{2}{3}} \pm \sqrt{1+2\sqrt{\frac{2}{3}}+\frac{2}{3} - 4\sqrt{\frac{2}{3}}}\right) \\ &= \frac{1}{2}\sqrt{\frac{2}{3}}\left(1+\sqrt{\frac{2}{3}} \pm \sqrt{\left(1-\sqrt{\frac{2}{3}}\right)^2}\right) \\ &= \frac{1}{2}\sqrt{\frac{2}{3}}\left(1+\sqrt{\frac{2}{3}} \pm \left(1-\sqrt{\frac{2}{3}}\right)\right) \end{aligned}$$

so

$$x = \sqrt{\frac{2}{3}} \quad \text{or} \quad x = \frac{2}{3}.$$

Question 4. $\frac{d}{dt}\sin t =$*?*

Answer 4.

$$\frac{d}{dt}\sin t = \cos t$$

Question 5. $\frac{d}{d\theta}\cos(2\theta) =$*?*

Answer 5.

$$\frac{d}{d\theta}\cos(2\theta) = -2\sin(2\theta)$$

Question 6. $\frac{d}{dx}(\sin(x)\cos(2x))$=*?*

Answer 6.

$$\frac{d}{dx}(\sin(x)\cos(2x)) = \cos(x)\cos(2x) - 2\sin(x)\sin(2x)$$

Date: September 22, 2015.

Question 7. $\frac{d}{dy}\tan y =$ *?*

Answer 7.

$$\frac{d}{dy}\tan y = \frac{1}{\cos^2(y)}$$

Question 8. $\frac{1}{2}\int_{-\alpha}^{+\alpha}\frac{d}{d\tau}(e^{4\tau^2})d\tau =$ *?*

Answer 8.

$$\frac{1}{2}\int_{-\alpha}^{+\alpha}\frac{d}{d\tau}(e^{4\tau^2})d\tau = \frac{1}{2}\left(e^{4\alpha^2} - e^{4(-\alpha)^2}\right) = 0$$

Question 9. $\frac{d}{d\zeta}\int_{-\zeta}^{+\zeta} se^s ds =$ *?*

Answer 9. We write $\xi = -\zeta$:

$$\begin{aligned}\frac{d}{d\zeta}\int_{-\zeta}^{+\zeta} se^s ds &= \frac{d}{d\zeta}\left(\int_0^{+\zeta} se^s ds - \int_0^{-\zeta} se^s ds\right)\\ &= \zeta e^\zeta - \frac{d\zeta}{d\xi}\frac{d}{d\xi}\int_0^\xi se^s ds\\ &= \zeta e^\zeta + \xi e^\xi\\ &= \zeta(e^\zeta - e^{-\zeta})\end{aligned}$$

Question 10. $\int_\gamma^\delta \frac{1}{\varphi} =$ *?, where* $\gamma, \delta > 0$.

Answer 10.

$$\int_\gamma^\delta \frac{1}{\varphi} = [\log|\varphi|]_\gamma^\delta = \log(\delta) - \log(\gamma) = \log\left(\frac{\delta}{\gamma}\right)$$