FIVE MINUTE REVIEW FOR WEEK 7.

Question 1. *Simplify* $\frac{1}{x+3} + \frac{1}{x^2-9}$.

Answer 1.
$$\frac{1}{x+3} + \frac{1}{x^2-9} = \frac{x-3}{(x-3)(x+3)} + \frac{1}{(x-3)(x+3)} = \frac{x-2}{(x-3)(x+3)}$$

Question 2. *Simplify* $1 + \frac{1}{1+\frac{1}{1+x}}$.

Answer 2.
$$1 + \frac{1}{1+\frac{1}{1+x}} = 1 + \frac{1+x}{1+x+1} = 1 + \frac{x+1}{x+2} = \frac{x+2+x+1}{x+2} = \frac{2x+3}{x+2}$$

Question 3. *State the Fundamental Theorem of Calculus.*

Answer 3. (a) Let f be integrable on $[a, b]$, and define a function F on $[a, b]$ by
$$F(x) = \int_a^x f(t)dt.$$

If f is continuous at $c \in [a, b]$, then F is differentiable at c, and
$$\frac{dF}{dx}(c) = f(c).$$

(b) If f is integrable on $[a, b]$ and $f(t) = \frac{dg}{dt}$ for some function $g(t)$, then
$$\int_a^b f(t)dt = g(b) - g(a).$$

Question 4. $\frac{d}{dx}(a^x) =$*?* *(*$a > 1$*)*

Answer 4.
$$\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{x\log(a)}) = \log(a)e^{x\log(a)} = \log(a)a^x$$

Question 5. $\frac{d}{d\theta}(\sin(\theta^2)) =$*?*

Answer 5.
$$\frac{d}{d\theta}(\sin(\theta^2)) = 2\theta\cos(\theta^2)$$

Question 6. $\frac{d}{ds}\int_0^{3s} f(\tau)g(\tau)d\tau =$*?*

Answer 6.
$$\frac{d}{ds}\int_0^{3s} f(\tau)g(\tau)d\tau = 3f(3s)g(3s)$$

Question 7. $\int_0^1 \frac{d\xi}{1+\xi^2} =$*?*

Answer 7. Let $\xi = \tan\theta$ so that $1 + \xi^2 = 1 + \tan^2\theta = \frac{1}{\cos^2\theta}$, $d\xi = \frac{1}{\cos^2\theta}d\theta$, and we are integrating from $\arctan(0) = 0$ to $\arctan(1) = \frac{\pi}{4}$. Then
$$\int_0^1 \frac{d\xi}{1+\xi^2} = \int_0^{\frac{\pi}{4}} \cos^2\theta \cdot \frac{d\theta}{\cos^2\theta} = \frac{\pi}{4}$$

Question 8. $\int_a^b (1+t)e^t dt =$*?*

Date: October 10, 2015.

Answer 8.

$$\int_a^b (1+t)e^t dt = \int_a^b \frac{d}{dt}(te^t)dt = be^b - ae^a$$

Question 9. *Simplify* $-\sum_{n\geq 0} x^{-n-1}$.

Answer 9.

$$-\sum_{n\geq 0} x^{-n-1} = -\frac{1}{x}\sum_{n\geq 0}(x^{-1})^n = -\frac{1}{x}\frac{1}{1-x^{-1}} = \frac{1}{1-x}$$

provided that $|x^{-1}| < 1$, i.e. $|x| > 1$ (the radius of convergence for the series).

Question 10. *Simplify* $\sum_{k\geq 0} \frac{t^{k+1}}{2^k k!}$.

Answer 10.

$$\sum_{k\geq 0} \frac{t^{k+1}}{2^k k!} = t\sum_{k\geq 0} \frac{\left(\frac{t}{2}\right)^k}{k!} = te^{\frac{t}{2}}$$