FIVE MINUTE REVIEW FOR WEEK 8.

Question 1. *Simplify* $\frac{x^2+x-2}{x^2-3x+2}$.

Answer 1.

$$\frac{x^2+x-2}{x^2-3x+2} = \frac{(x+2)(x-1)}{(x-2)(x-1)} = \frac{x+2}{x-2}$$

Question 2. *Simplify* $\sqrt{16a^4b^3}$.

Answer 2.

$$\sqrt{16a^4b^3} = 4a^2|b|\sqrt{b}$$

Question 3. $\frac{d}{dx}\left(3\sin^2(x)+\cos(2x)\right) = ?$

Answer 3.

$$\frac{d}{dx}\left(3\sin^2(x)+\cos(2x)\right) = \frac{d}{dx}\left(3\sin^2(x)+1-2\sin^2(x)\right) = \frac{d}{dx}(\sin^2(x)) = 2\sin(x)\cos(x) = \sin(2x)$$

Question 4. $\frac{d}{d\theta}\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right) = ?$

Answer 4.

$$\frac{d}{d\theta}\left(\frac{e^{i\theta}-e^{-i\theta}}{2i}\right) = \frac{d}{d\theta}(\sin\theta) = \cos\theta$$

Question 5. $\frac{d}{dt}\left(\log\left(\frac{1}{t}\right)\right) = ?$ *(*$t > 0$*)*

Answer 5.

$$\frac{d}{dt}\left(\log\left(\frac{1}{t}\right)\right) = -\frac{d}{dt}\log(t) = -\frac{1}{t}$$

Question 6. $\frac{d}{d\zeta}\left(\log\left(e^{\zeta}+\zeta^2\right)\right) = ?$ *(*$\zeta \neq 0$*)*

Answer 6.

$$\frac{d}{d\zeta}\left(\log\left(e^{\zeta}+\zeta^2\right)\right) = \frac{1}{e^{\zeta}+\zeta^2}\frac{d}{d\zeta}(e^{\zeta}+\zeta^2) = \frac{e^{\zeta}+2\zeta}{e^{\zeta}+\zeta^2}$$

Question 7. $\int_{\frac{\pi}{4}}^{0} \frac{ds}{\cos^2(s)} = ?$

Answer 7.

$$\int_{\frac{\pi}{4}}^{0} \frac{ds}{\cos^2(s)} = -\int_0^{\frac{\pi}{4}} \frac{d(\tan(s))}{ds}ds = -\left(\tan\left(\frac{\pi}{4}\right)-\tan(0)\right) = -(1-0) = -1$$

Question 8. $\frac{d}{d\xi}\int_0^{e^{\xi}} \sin^3(\varphi)d\varphi = ?$

Answer 8. Let $u = e^{\xi}$. Then

$$\frac{d}{d\xi}\int_0^{e^{\xi}} \sin^3(\varphi)d\varphi = e^{\xi}\frac{d}{du}\int_0^{u} \sin^3(\varphi)d\varphi = e^{\xi}\sin^3(u) = e^{\xi}\sin^3(e^{\xi})$$

Question 9. *Simplify (and find radius of convergence of* $\sum_{n\geq 1} nx^{n-1}$.

Answer 9.

$$\sum_{n\geq 1} nx^{n-1} = \sum_{n\geq 0}\frac{d}{dx}(x^n) = \frac{d}{dx}\sum_{n\geq 0}x^n = \frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$$

where $|x| < 1$ (since this is the radius of convergence of the geometric series).

Date: October 19, 2015.

Question 10. *Simplify* $\dfrac{d}{dt}\left(\displaystyle\sum_{m\geq 0}(-1)^m\frac{t^m}{m!}\right)$.

Answer 10.

$$\frac{d}{dt}\left(\sum_{m\geq 0}(-1)^m\frac{t^m}{m!}\right)=\frac{d}{dt}\left(e^{-t}\right)=-e^{-t}$$