BENNETT DIFFERENTIAL EQUATIONS PRIZE EXAM May 8 2018

Name:_____ **UT EID:**_____

Differential Equations Course:_____ **When?**_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Submit your solutions on the sheets provided, with your name on each sheet. No calculators allowed. You must justify your claims.

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1. Find the general solution of $x^4y'' + 5x^3y' + 4x^2y = 1$.

2. Sketch the solution to the differential equation

$$\frac{dy}{dx} = y^4 + 4 \qquad y(3) = 0$$

Identify any critical points and inflection points, and explain why there are or are not any horizontal or vertical asymptotes.

3. Solve the differential equation

$$(4xy + 2y^2 + 2x)\frac{dy}{dx} = x^2 + 2xy + 3y^2 + 2y \qquad y(1) = -2$$

Hint: there is an integrating factor μ for which $\partial\mu/\partial x = \partial\mu/\partial y$.

4. Solve the system $\dfrac{dx}{dt} = y(x+y)^5, \qquad \dfrac{dy}{dt} = x(x+y)^5, \qquad x(0) = 1, \quad y(0) = 0$
(*Hint:* Add and subtract.)

5. The *biharmonic equation* from continuum mechanics is the fourth-order linear partial differential equation $u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0$. For partial credit, find a nonzero solution $u(x, y)$ to this equation. For full credit, find a non-polynomial solution. For extra credit, find an infinite-dimensional vector space of solutions.

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/` .