BENNETT LINEAR ALGEBRA PRIZE EXAM **Dec 11 2018**

Name:________________________ **UT EID:**____________________
Linear Algebra Course:________ **When?**________ **Instructor:**____________
Permanent Mailing Address:____________________________________

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E-mail address:____
College (Natural Sciences, Engineering, etc.)______________________

Submit your solutions on the sheets provided, with your name on each sheet. No calculators allowed. You must justify your claims.

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1. Find a polynomial $f(x)$ which has the same values as $g(x) = \frac{120}{x}$ for $x = 1, 2, 3, 4, 5$. (That is, we need $f(1) = 120$, $f(2) = 60$, etc.)

2. Suppose A and B are square matrices of the same size, and that $ABABA = I$.

(a) Explain why A is invertible.

(b) Show that $AB = BA$.

3. The exponential function is defined for square matrices A by the usual power series:

$$e^A = I + A + \frac{1}{2}A^2 + \ldots = \sum_{n=0}^{\infty} \frac{1}{n!}A^n$$

Compute e^A when $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$.

4. A linear transformation $L : \mathbf{R}^n \to \mathbf{R}^n$ is called a *projection* if $L(L(v)) = L(v)$ for each $v \in \mathbf{R}^n$. For example the function $L(x, y, z) = (2y + 3z, y, z)$ is a projection in $\mathbf{R}^3$.

Show that the only possible eigenvalues of a projection L are 0 and 1.

5. Find an invertible matrix P for which $PAP^{-1} = B$ where

$$A = \begin{pmatrix} 1 & 2018 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1 & 41 \\ 0 & 1 \end{pmatrix}$$

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/`

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