BENNETT DIFFERENTIAL EQUATION PRIZE EXAM May 11 2021

Submit your solutions *with all work shown*, **by 8pm (Austin time) as an email attachment to rusin@math.utexas.edu. During the exam you must abide the rules previously sent via email.**

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1. Two functions f and g are said to be *inverses of each other* if $f(g(t)) = t$ for every t in some interval, and $g(f(u)) = u$ for every u in another interval. For example the logarithm and exponential functions are inverses of each other, as are the cosine function and any branch of the arc-cosine function.

Find a differentiable function f for which f and f' are inverses of each other.

2. Find the general solution of $yy'' + (y')^2 = 5(y')^3$

3. An object moves along the real number line; its position $x(t)$ at time t satisfies

$$x''(t) + 4x(t) = x(t)\,(x'(t))^2$$

Show that if $x(0) = 1$ and $x'(0) = 0$ then the object follows an oscillating trajectory, but that some other initial conditions do not lead to oscillation.

4. Find a solution $u(x, t)$ valid for all x in $[0, \pi]$ and all $t \geq 0$ to

$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

with boundary conditions $u(0, t) = u(\pi, t) = 0$ for all $t \geq 0$ and initial conditions $u(x, 0) = 0$ and $\frac{\partial u}{\partial t}(x, 0) = 14 \sin(5x)$ for all $x \in [0, \pi]$.

5. Solve the system

$$x'(t)+2y'(t) = 8x(t)+14y(t) \qquad x'(t)+y'(t) = -7x(t)-13y(t) \qquad x(0) = 13, y(0) = -8$$

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/` .