ALBERT A. BENNETT CALCULUS PRIZE EXAM   DATE 12/05

Name:______________________________ **UT EID**___________________________
Present Calculus Course:___________ **Instructor**____________________________
Permanent Mailing Address:____

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School (Nat'l Sciences, Engineering, *etc*)**:**_____________________________

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

1. Evaluate the following limits:

(i) $\lim_{x\to 0} x\sin(1/x)$

(ii) $\lim_{x\to\infty} x\sin(1/x)$

(iii) $\lim_{x\to 1} \frac{x^3-3x+2}{x^5-5x+4}$

(iv) $\lim_{x\to 0} \frac{x^3-3x+2}{x^5-5x+4}$

(v) $\lim_{x\to 0^+} (1+\frac{1}{x})^x$

2. Suppose we define, for a function $f(x)$,

$$f^*(x) = \lim_{h\to 0} \frac{f(x+h)-f(x-h)}{2h}$$

for all x for which the limit exists. This looks quite similar to the definition of the derivative, $f'(x)$ since it is the limit of [the difference of f at 2 values near x divided by the difference between the values] as the values both approach x. Is this definition equivalent to the definition of $f'(x)$? If your answer is yes, justify this; if it is no, give examples to show why the definition is not equivalent.

3. Is there a value of d such that x^3-3x+d has 2 roots in the interval (0,1)?

What if we change the interval to $(0, 1.75)$?

4. Suppose x_n are real numbers with $0 < x_1 < 1$, and $x_{n+1} = x_n(1-x_n)$. Find $\lim_{n\to\infty} x_n$ and justify all reasoning completely.

**For Extra Credit:* What do you think about the convergence (or divergence) of $\sum_{n=1}^{\infty} x_n$? (*Note*: This is much harder than the first part of the question, so you probably shouldn't work on it till you have worked on everything else and have extra time.)

5. Find the minimum and maximum of the function $f(x, y) = \sin(xy)$ restricted to the set $x^2 + y^2 = 1$ in $\mathbf{R}^2$.

Do the same for $f(x, y) = \sin(xy)$ restricted to $x^2 + y^2 = 4$.