ALBERT A. BENNETT CALCULUS PRIZE EXAM Dec 5 2010

Name:	UT EID:
Present Calculus Course:	Instructor:
Permanent Mailing Address:	
E-mail address:	

School (Natural Sciences, Engineering, etc.)

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

1. Evaluate
$$\lim_{x \to \infty} \frac{\sqrt{x^3 - x^2 + 3x}}{\sqrt{x^3} - \sqrt{x^2} + \sqrt{3x}}.$$

2. Determine whether these series converge or diverge. (Be sure to justify your answer.)

(a)
$$\sum_{n=2}^{\infty} \frac{n^8 - 1}{n^9 - 1}$$
 (b) $\sum_{n=2}^{\infty} \frac{1}{\ln(n!)}$

3. Compute
$$\lim_{x \to 0} \frac{\cos(2x) + 2\sin(x^2) - 1}{x^4}$$
.

4. The four points

$$A = (-6, -2, 3),$$
 $B = (-6, 8, 3),$ $C = (-7, 5, 3),$ $D = (4, -6, 5)$

are all equally far from a point P. Find P.

5. Compute the minimum value of the function

$$f(u,v) = \left(u-v\right)^2 + \left((3-u) - \left(\frac{5}{v}\right)\right)^2$$

on the region where v > 0.