## ALBERT A. BENNETT CALCULUS PRIZE EXAM Dec 12 2017

Name:	UT EID:
Present Calculus Course:	Instructor:
Permanent Mailing Address:	
E-mail address:	
College (Natural Sciences, Engineering	, etc.)

Submit your solutions with all work shown on the sheets provided. No calculators allowed. (Suggestion: Do preliminary work on scratch paper that you don't submit; write up final solutions neatly and in order; write your name on all pages submitted.)

- 1. The equation  $y^3 + x^2y + 2x^3 3x^2 + 1 = 0$  defines a curve in the plane. We view this curve as the graph of a function y = f(x). Find all the critical points of this function and classify them as local maxima or local minima.
- 2. Show that  $I = \int_{1}^{2} \frac{1}{4+x^4} dx$  lies between  $\frac{1}{20}$  and  $\frac{7}{24}$ . Five points extra credit goes to the contestant who finds the smallest such interval containing the value of I.
- **3.** Does the following series converge? (Why or why not?)

$$\sum_{k=0}^{\infty} \left( 3 \cdot \frac{\ln(4k+2)}{4k+2} - \frac{\ln(4k+3)}{4k+3} - \frac{\ln(4k+4)}{4k+4} - \frac{\ln(4k+5)}{4k+5} \right)$$
$$= 3 \cdot \frac{\ln 2}{2} - \frac{\ln 3}{3} - \frac{\ln 4}{4} - \frac{\ln 5}{5} + 3 \cdot \frac{\ln 6}{6} - \frac{\ln 7}{7} - \frac{\ln 8}{8} - \frac{\ln 9}{9} + 3 \cdot \frac{\ln 10}{10} - \cdots$$

4. Compute the limit or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \left(\cos(x+y)\right)^{\cot(x^2-xy+y^2)}$$

5. Compute

$$\int_{y=0}^{1} \left( \int_{x=0}^{1} \frac{x-y}{(x+y)^3} dx \right) dy \quad \text{and} \quad \int_{x=0}^{1} \left( \int_{y=0}^{1} \frac{x-y}{(x+y)^3} dy \right) dx$$

Answers will soon appear at http://www.math.utexas.edu/users/rusin/Bennett/ .