

Name: _____ UT EID: _____

Present Calculus Course: _____ Instructor: _____

Permanent Mailing Address: _____

E-mail address: _____

College (Natural Sciences, Engineering, etc.) _____

Submit your solutions *with all work shown on the sheets provided. No calculators allowed.* (Suggestion: Work first on scrap paper that you don't submit; write up final solutions neatly and in order, with your name on all pages submitted.)

1. The equation $x^y = y^x$ describes a curve in the first quadrant of the plane containing the point $P = (4, 2)$. Compute the slope of the line that is tangent to this curve at P .
Some extra credit will be given for a good sketch of the graph of this curve.

2. Determine whether this series is convergent or divergent:

$$\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln(\ln(n))}}$$

3. Compute $\int_0^{\pi/4} \frac{1}{\cos(x) + \sin(x)} dx$.

4. A *wedding ring* is the three-dimensional solid that remains after drilling a cylindrical hole through the center of a sphere. Compute, with proof, the volume of metal in a metallic wedding ring that is 6mm tall when it rests on a table, as a function of the radius r of the hole that has been drilled.

5. The curve parameterized by $x(t) = \cos^3(t)$, $y(t) = \sin^3(t)$, $z(t) = \cos(2t)$ passes through the point $(1, 0, 1)$ when $t = 0$ and passes through the point $(0, 1, -1)$ when $t = \pi/2$, having traversed a path of length $5/2$. (You don't have to prove this.) What point will it pass through after having traversed a length of exactly 1 ?

Answers will soon appear at <http://www.math.utexas.edu/users/rusin/Bennett/> .