ALBERT A. BENNETT CALCULUS PRIZE EXAM 5/3/08

Name:______________________________ **UT EID:**__________________________

Present Calculus Course:____________ **Instructor:**______________________________

Permanent Mailing Address:____

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E-mail address:____

School (Natural Sciences, Engineering, etc.)____________________________________

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Let $f(x) = \int_0^x \cos(t^2)\,dt$. Write the Maclaurin series (Taylor series centered at 0) for each of the following functions of x.

(i)	$\cos(x)$	(iii)	$f(x)$
(ii)	$\cos(x^2)$	(iv)	$g(x) = f(x^2)$

2. Find the equations of all lines which are tangent to the curve $y = x^3 - x$ and are perpendicular to the line $y = 4x + 5$.

3. Let a curve be given by the parametric equations

$$x = e^t \sin t - e^t \cos t$$

$$y = e^t \sin t + e^t \cos t$$

Find the arclength of the curve from $t = 0$ to $t = \ln(2)$.

4. Suppose that x and y are given as functions of s and t by the equations

$$x = e^{st}\ , \qquad y = s^2t^3$$

Suppose also that s is a function of t such that $ds/dt = (1 + t^3)^{-1}$ Then y can be regarded as a function of x. Compute dy/dx in terms of s and t.

5. Let $f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0\ , \\ 0 & \text{if } x = 0\ . \end{cases}$

(i) Show that $\lim_{x \to 0} f'(x)$ does not exist.

(ii) Using the definition of the derivative, show that $f'(0) = 0$.