ALBERT A. BENNETT CALCULUS PRIZE EXAM 5/9/09

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Find the largest possible volume for a right circular cone inscribed in a sphere of radius one. Recall that the volume of a right circular cone is $1/3\,hA$ where h is the height and A is the area of the base.

2. (i) Find the distance from the point $(3, 6, 5)$ to the plane $x + 2y + 3z = 2$.

(ii) Find the distance between the parallel planes $x+2y+3z = 2.$ and $x+2y+3z = 0.$

In working parts (i) and (ii) do not use the general formula for the distance from a point to a plane for the distance between two parallel planes unless you prove it.

3. Compute the sum $\displaystyle\sum_{n=0}^{\infty}(3+(-1)^n)^{-n}$.

(Hint: Write the first several terms of the series.)

4. Find the equation of each line which passes through the origin and is tangent to the curve $y = x^4 + x^3 - x^2 + 2x$ at some point.

5. Compute $\displaystyle\int_{-1}^{0} \frac{1}{(x+2)^3\sqrt{x^2+4x+3}}\,dx.$