ALBERT A. BENNETT CALCULUS PRIZE EXAM **5/8/10**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Find the equation of the plane that passes through the points (1,2,2) and $(-1, 1, 3)$ and is parallel to the line $x = 1 + 2t$, $y = 4 - t$, $z = 3t$.

2. Let $f(x) = \sin(x^3)$. Find the 99^{th} derivative of f evaluated at 0. That is, find $f^{(99)}(0)$.

3. Find the point on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ that is farthest from the line $2x + y = 10$.

4. Let C_1 be the solid cylinder in 3-dimensional space consisting of all points whose distance from the x-axis is not greater than 6. Let C_2 be the solid cylinder consisting of all points whose distance from the y-axis is not greater than 6. If V is the intersection of C_1 and C_2, find the volume of V. (Hint: If T is a plane parallel to the xy-plane, what does the intersection of T with V look like?)

5. Let f be a 3^{rd} degree polynomial. That is, $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$. Show that there is at least one number x_0 such that $f(x_0) = 0$.