ALBERT A. BENNETT CALCULUS PRIZE EXAM **May 7 2011**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

School (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Determine whether these series converge or diverge. (Be sure to justify your answer.)

$$(a)\ \sum_{n=1}^{\infty} \frac{1}{(3n-2)^{n+(1/2)}} \qquad (b)\ \sum_{n=1}^{\infty} \frac{(-1)^{n(n-1)/2}}{n} = 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

2. Compute the following limit, or show that it does not exist:

$$\lim_{x \to 0} \frac{x^3 \sin(\frac{1}{x})}{\ln(1+x^2)}$$

3. Compute the first three terms $a_0 + a_1 x + a_2 x^2$ of the Maclaurin series (i.e. the Taylor series at 0) for

$$f(x) = \frac{5x-7}{(x-1)(x-2)}$$

4. Find a point which is equidistant from all four planes

$$x = 0 \qquad y = 0 \qquad z = 0 \qquad 2x + 3y + 6z = 36.$$

5. Find all the critical points of the function below, and state whether they are local minima, local maxima, or saddle points:

$$f(x,y) = 1 - (x^2-1)^2 - (x^2 y - x - 1)^2.$$