ALBERT A. BENNETT CALCULUS PRIZE EXAM **May 4 2014**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

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1. Compute the integral or explain why the integral does not converge:

$$\int_{-2}^{+1} \frac{(10+4x)}{(5x+x^2)^3}\,dx$$

2. Compute the sum $\displaystyle\sum_{n=0}^{\infty} \frac{6\cdot 3^n - 2^{n+3} + 3}{4^n}$ or explain why the series does not converge.

3. Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x-y}{\sin(x)-\sin(y)}$ or explain why the limit does not exist.

4. Let A be an $a \times b \times c$ brick in R^3. Then let B be the set of points in R^3 which are outside of A but whose distance from A is less than 1. What is the volume of B?

5. Let C be the curve defined by the equation $y^2 = 2x(x+2)(x+8)$, that is,

$$C = \left\{(x,y) \,|\, y^2 = 2x(x+2)(x+8)\right\}.$$

Find all lines that are tangent to the curve C and which also pass through the origin.

Answers will be posted to `http://www.math.utexas.edu/users/rusin/Bennett/` shortly.