Name:_____   UT EID:_____

Present Calculus Course:_____   Instructor:_____

Permanent Mailing Address:_____

_____

E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. No calculators allowed. (Suggestion: Do preliminary work on scratch paper that you don't turn in; write up final solutions neatly and in order; write your name on all pages turned in.)

1. How many real solutions of the equation $e^x + x = 2$ are there? (Hint: first decide how many solutions there are inside the interval $[0, 1]$.)

2. If the line $y = mx$ is tangent to the curve $y = e^{ax}$, what is the relationship between m and a?

3. The *greatest integer function* is the function whose value at a number x is the largest integer n which is less than or equal to x. This value is denoted by $\lfloor x \rfloor$. For example

$$\lfloor 3 \rfloor = \lfloor \pi \rfloor = \lfloor \frac{7}{2} \rfloor = \lfloor 3.99997 \rfloor = 3 \qquad \text{and} \qquad \lfloor 4 \rfloor = 4$$

   Compute $\int_0^\infty \lfloor x \rfloor e^{-x}\, dx$.

4. For what values of x does this series converge?

$$\sum_{n=1}^\infty \frac{n!\, x^{2n}}{n^n\, (1 + x^{2n})}$$

5. I have 6 meters of fencing which I wish to use to enclose as much area as I can in two completely disjoint regions — one is a square and the other is an equilateral triangle. How much area can I enclose in two such regions having a combined perimeter of 6 meters?

Answers will be posted to `http://www.math.utexas.edu/users/rusin/Bennett/` shortly.