ALBERT A BENNETT CALCULUS PRIZE EXAM **May 13 2019**

Name:_____ **UT EID:**_____

Present Calculus Course:_____ **Instructor:**_____

Permanent Mailing Address:_____

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E-mail address:_____

College (Natural Sciences, Engineering, etc.)_____

Show all work in your solutions; turn in your solutions on the sheets provided. No calculators allowed. (Suggestion: Work first on scrap paper that you don't submit; write up final solutions neatly and in order, with your name on all pages submitted.)

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1. For which real numbers r does this limit exist?

$$\lim_{x \to 0^+} x^r \ln(x)$$

2. Find an antiderivative of $\cos^4(x) - \sin^4(x)$.

3. Do these series converge or diverge? Explain.

$$(A)\ \sum_{n=1}^{\infty} \sin\left(\frac{\cos(n)}{n^2}\right) \qquad (B)\ \sum_{n=1}^{\infty} \cos\left(\frac{\sin(n)}{n^2}\right)$$

4. Compute $\dfrac{dy}{dx}$ where $y = \arcsin(2uv)$, $u = \cos(x)$, and $v = \sin(x)$. You may assume that $x \in [0, \pi/4]$.

5. A 1-meter-long rod is lying at the base of a 5-meter-tall streetlamp. The rod is oriented north-south. A runner raises the rod to a height of 2 meters and heads east at a rate of 4 meters per second, always keeping the rod perpendicular to his path, level to the ground, and at a height of 2 meters. The rod will then produce a moving shadow on the ground. How rapidly does the width of the rod's shadow increase as the runner moves eastward?

Answers will soon appear at `http://www.math.utexas.edu/users/rusin/Bennett/` .