MATH 361K – HOMEWORK ASSIGNMENT 1

Due Thursday, Jan 29, 2009

Please write clearly, and staple your work !

1. Problems

Assume that we want to prove or disprove the statement "X implies Y" (abbreviated by "$X \Rightarrow Y$"). Which of the following are correct ?

(i) We can instead prove the statement "if Y is not satisfied, then X is also not satisfied". This implies "$X \Rightarrow Y$".
(ii) We can try to find an example where X does not imply Y. If we succeed, we can conclude that the statement "$X \Rightarrow Y$" is wrong.
(iii) We can instead prove the statement "if X is not satisfied, then Y is also not satisfied". This implies "$X \Rightarrow Y$".

2. Problem

(a) Let A, B be sets. Prove that $A \Delta B = (A \cup B) \setminus (A \cap B)$.
(b) Let $A, B \subset U$ be sets. Prove that $C(A \cup B) = C(A) \cap C(B)$.

3. Problem

Let A and B be sets and assume that $f : A \to B$ is bijective.

(a) Prove that if A has cardinality n, then so does B.
(b) Is it possible that there exists a function $h : B \to A$ such that $f(h(b)) = b$ for all $b \in B$, but $h(f(a)) \neq a$ for some $a \in A$? (You may use the fact that f^{-1} exists.)
(c) How many different bijections $f : A \to B$ exist if both A and B have cardinality n ?

4. Problem

Assume that $x, y \geq 0$ are real numbers. Prove by mathematical induction that for all $n \in \mathbb{N}$,

$$(x^n + y^n)^{\frac{1}{n}} \leq x + y \,.$$

5. Problem

Assume that $r_1, r_2, \ldots, r_{70000}$ is a list of 70000 real numbers, $0 < r_j < 1$, given in decimals. How would you use Cantor's diagonal procedure to easily find nine real numbers, also between 0 and 1, that are not in this list ?