

MATH 361K – HOMEWORK ASSIGNMENT 10

Due Thursday, April 23, 2009

Please write clearly, and staple your work !

1. PROBLEM

Let $I \subseteq \mathbb{R}$ be an interval, and $f : I \rightarrow \mathbb{R}$ differentiable on I . Prove that if $f' > 0$ on I , then f is strictly increasing on I .

2. PROBLEMS

Use the mean value theorem to prove that $|\sin x - \sin y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

3. PROBLEMS

Give an example of a uniformly continuous function on $[0, 1]$ that is differentiable on $(0, 1)$ but whose derivative is not bounded on $(0, 1)$.

4. PROBLEM

Use the mean value theorem to prove that $\frac{x-1}{x} < \ln x < x - 1$ for $x > 1$ (using that $(\ln x)' = \frac{1}{x}$ for $x > 0$).

5. PROBLEM

Let $f : [a, b] \rightarrow \mathbb{R}$ be continuous on $[a, b]$ and differentiable on (a, b) . Show that if $\lim_{x \rightarrow a} f'(x) = A$, then $f'(a)$ exists and equals A . (Use the definition of $f'(a)$ and the mean value theorem.)