MATH 361K – HOMEWORK ASSIGNMENT 3

Due Thursday, Feb 12, 2009

Please write clearly, and staple your work !

1. PROBLEMS

Assume that (x_n) and (y_n) are convergent, real sequences with limits $x = \lim x_n$ and $y = \lim y_n$ (where for brevity, $\lim \equiv \lim_{n \to \infty}$).

  (a) Prove that $\lim(x_n + y_n) = x + y$.

  (b) Using that $\lim(x_n y_n) = xy$, prove that $\lim x_n^a = x^a$ for any $a \in \mathbb{N}$.

2. PROBLEM

Consider the sequence (x_n) which is recursively defined by $x_1 = 1$ and

$$x_{n+1} = r x_n \,.$$

For which values of $r \in \mathbb{R}$, $r > 0$, is (x_n) convergent, and what is its limit ?

3. PROBLEM

Consider the sequence (x_n) which is recursively defined by $x_1 = 4$ and

$$x_{n+1} = 2 - \frac{1}{x_n} \,.$$

Prove that it is bounded and monotone, and find its limit.

4. PROBLEM

Assume that (x_n) is a bounded real sequence, with $a < x_n < b$ for all n. Which of the following is true (give a proof or a counterexample) ?

  (a) It follows that (x_n) is convergent.

  (b) It follows that (x_n) always has precisely one convergent subsequence.

  (c) It follows that (x_n) always has more than one convergent subsequence.

  (d) If (x_n) is convergent, and $x = \lim x_n$, then every subsequence is also convergent. In particular, every subsequence converges to x.

  (e) If (x_n) has two convergent subsequences with different limits, then (x_n) itself is not convergent.

  (f) If (x_n) has two convergent subsequences where one converges to a and the other to b, then (x_n) also has a convergent subsequence with a limit c where $a < c < b$.