MATH 361K – HOMEWORK ASSIGNMENT 4

Due Thursday, Feb 19, 2009

Please write clearly, and staple your work !

1. PROBLEM

Prove the Bolzano-Weierstrass theorem ("every bounded sequence (x_n) has a convergent subsequence") using the monotone subsequence theorem.

2. PROBLEMS

Prove that the sequence (x_n) does *not* converge to x if and only if there exists a subsequence (x_{n_k}) and some $\epsilon_0 > 0$ such that $|x - x_{n_k}| > \epsilon_0$ for all $k \in \mathbb{N}$.

3. PROBLEMS

Prove that $(\frac{1}{2^n})$ is a Cauchy sequence, and find its limit.

4. PROBLEM

Prove that a bounded, monotone sequence is a Cauchy sequence.

5. PROBLEM

A sequence (x_n) is called *contractive* if $|x_{n+1} - x_n| \leq C|x_n - x_{n-1}|$ for some constant $0 < C < 1$, and all $n \in \mathbb{N}$. Prove that a contractive sequence is a Cauchy sequence.