# MATH 361K – HOMEWORK ASSIGNMENT 5

Due Thursday, March 5, 2009

**Please write clearly, and staple your work !**

## 1. Problem

If $\sum x_n$ with $x_n > 0$ is convergent, then is $\sum x_n^2$ always convergent ? Either prove it, or give a counterexample.

## 2. Problems

Assume that the series $\sum x_n$, with $x_n > 0$, is convergent. Defining $y_n := \frac{1}{n}(x_1 + \cdots + x_n)$, prove that $\sum y_n$ is always divergent.

## 3. Problems

Assume $\sum_{n=1}^{\infty} x_n$ where $(x_n)$ with $x_n > 0$ is a strictly decreasing sequence of real numbers. let $s_n := \sum_{k=1}^{n} x_n$ denote the $n$-th partial sum. Prove that

$$\frac{1}{2}(x_1 + 2x_2 + 4x_4 + \cdots + 2^n x_{2^n}) \le s_{2^n} \le (x_1 + 2x_2 + \cdots + 2^{n-1} x_{2^{n-1}}) + x_{2^n} .$$

Use these inequalities to prove that $\sum_{n=1}^{\infty} x_n$ converges if and only if $\sum 2^n x_{2^n}$ converges. This is the so-called *Cauchy condensation test.*

## 4. Problem

Use the Cauchy condensation test to prove that the $p$-series $\sum \frac{1}{n^p}$ converges for all $p > 1$.

## 5. Problem

Use the Cauchy condensation test to prove that $\sum \frac{1}{n \ln n}$ diverges.