

## MATH 361K – HOMEWORK ASSIGNMENT 6

Due Thursday, March 12, 2009

**Please write clearly, and staple your work !**

### 1. PROBLEM

Let  $f : A \rightarrow \mathbb{R}$ , and let  $c$  be a cluster point of  $A$ . Prove that the following are equivalent:

- (1)  $\lim_{x \rightarrow c} f(x) = L$ .
- (2) Given any  $\epsilon$ -neighborhood  $V_\epsilon(L)$  of  $L$ , there exists a  $\delta$ -neighborhood  $V_\delta(c)$  of  $c$  such that if  $x \neq c$  is any point in  $V_\delta(c) \cap A$ , then  $f(x)$  belongs to  $V_\epsilon(L)$ .

### 2. PROBLEMS

Prove that the limit  $\lim_{x \rightarrow 0} \sin(\frac{1}{x^2})$  does not exist.

### 3. PROBLEMS

Prove that  $\lim_{x \rightarrow c} \sqrt{x} = \sqrt{c}$  for any  $c > 0$ .

### 4. PROBLEM

Let  $f : A \rightarrow \mathbb{R}$ , and let  $c$  be a cluster point of  $A$ . Prove that  $f$  does *not* have a limit at  $c$  if and only if there exists a sequence  $(x_n)$  in  $A$ , with  $x_n \neq c$  for all  $n \in \mathbb{N}$ , such that  $(x_n)$  converges to  $c$ , but  $(f(x_n))$  does *not* converge in  $\mathbb{R}$ .