Due Thursday, March 12, 2009

Please write clearly, and staple your work!

1. **Problem**

Let $f : A \to \mathbb{R}$, and let $c$ be a cluster point of $A$. Prove that the following are equivalent:

1. $\lim_{x \to c} f(x) = L$.
2. Given any $\epsilon$-neighborhood $V_\epsilon(L)$ of $L$, there exists a $\delta$-neighborhood $V_\delta(c)$ of $c$ such that if $x \neq c$ is any point in $V_\delta(c) \cap A$, then $f(x)$ belongs to $V_\epsilon(L)$.

2. **Problems**

Prove that the limit $\lim_{x \to 0} \sin\left(\frac{1}{x^2}\right)$ does not exist.

3. **Problems**

Prove that $\lim_{x \to c} \sqrt{x} = \sqrt{c}$ for any $c > 0$.

4. **Problem**

Let $f : A \to \mathbb{R}$, and let $c$ be a cluster point of $A$. Prove that $f$ does not have a limit at $c$ if and only if there exists a sequence $(x_n)$ in $A$, with $x_n \neq c$ for all $n \in \mathbb{N}$, such that $(x_n)$ converges to $c$, but $(f(x_n))$ does not converge in $\mathbb{R}$. 