

## MATH 361K – HOMEWORK ASSIGNMENT 7

Due Thursday, March 26, 2009

**Please write clearly, and staple your work !**

### 1. PROBLEM

Consider the function  $f(x) = e^{\frac{1}{x}}$ . Does it have left and right limits at  $x = 0$  ? If yes, determine them. Is  $f$  continuous at  $x = 0$  or not ? Prove your answer.

### 2. PROBLEMS

Let  $f$  be defined for all  $x \in \mathbb{R}$ ,  $x \neq 1$ , by  $f(x) = \frac{x^2 - 2x + 1}{x - 1}$ . Can  $f$  be defined at  $x = 1$  in such a way that  $f$  is continuous at this point ?

### 3. PROBLEMS

Let  $K > 0$  and let  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfy the condition  $|f(x) - f(y)| < K|x - y|$  for all  $x, y \in \mathbb{R}$ . Prove that  $f$  is continuous at every point  $x \in \mathbb{R}$ .

### 4. PROBLEM

Assume that  $f : \mathbb{R} \rightarrow \mathbb{R}$  is continuous on  $\mathbb{R}$ , and that  $f(r) = 0$  for every rational number  $r$ . Prove that  $f(x) = 0$  for all  $x \in \mathbb{R}$ .