MATH 361K – HOMEWORK ASSIGNMENT 8

Due Thursday, April 2, 2009

Please write clearly, and staple your work !

1. PROBLEM

We have learned that if f is continuous on the closed, bounded interval $I = [a, b]$, then $f(I)$ is also a closed, bounded interval. If we now assume that f is continuous on the open interval $I = (a, b)$, is it correct that $f(I)$ is open or not ? (Hint: Think of the function $f(x) = x^2$ on $(-1, 1)$.)

2. PROBLEMS

Prove that the equation $x = \cos x$ has a solution on $[0, \frac{\pi}{2}]$.

3. PROBLEMS

(a) Is the function $f(x) = x^2$ uniformly continuous on $[0, 1]$? How about $(0, 1)$?

(b) Is the function $g(x) = \frac{1}{x}$ uniformly continuous on $(0, 1]$? How about $[\frac{1}{100}, 1]$?

4. PROBLEM

Use the bisection method to find a solution of the equation $x = \cos x$ on $[0, \frac{\pi}{2}]$ with error less than 0.01.