MATH 361K – REVIEW 1

Will not be graded, but will be discussed on Tue, Feb 24, in class.

1. Problem

(a) Prove that if a monotone sequence (x_n) has a convergent subsequence (x_{n_k}), then (x_n) is also convergent.

(b) Assume that (x_n) is a bounded, increasing sequence. Assume that $\sup\{x_n\}$ is contained in the set $\{x_n\} \subset \mathbb{R}$. What does this mean for the sequence (x_n) ? What does it mean for the sequence (x_n) if this is not the case ?

2. Problems

Assume that (x_n) is properly divergent. Prove that it has no convergent subsequence. Is every unbounded divergent sequence a properly divergent sequence ? Is it true that any unbounded divergent sequence (x_n) has no convergent subsequence ?

3. Problems

Do the sequences $(\frac{n^2}{2^n})$ and $(\frac{2^n}{n!})$ converge ?

4. Problem

Consider finite sequences $(s_1, s_2, \ldots, s_n)$, all with n entries, and $s_j = 0$ or 1. Assume we list all possible 2^n different sequences of this form. Then it seems that Cantor's diagonal procedure cannot produce a 2^n+1-th sequence of this form that is different from all those already in the list. Is this true ? If yes, how is this possible ?

5. Problem

Consider two sequences (x_n) and (y_n) where $\lim x_n = \infty$ while $\lim y_n = 0$. Is it ever true that $\lim x_n y_n = (\lim x_n)(\lim y_n)$? If yes, give an example. If no, explain why not.