MATH 361K – REVIEW 3

Will not be graded, but will be discussed on Thu, May 7, in class.

1. Problem

Assume that $f : I = (a,b) \to J = (c,d)$ is differentiable, and $f'(x) \neq 0$ for $x \in (a,b)$. Furthermore, assume that f is bijective and thus has an inverse, $f^{-1} : J \to I$. This means that $(f \circ f^{-1})(y) = y$ for all $y \in J$, and $(f^{-1} \circ f)(x) = x$ for all $x \in I$. Prove that $(f^{-1})'(y) = 1/f'(f^{-1}(y))$ for all $y \in J$, using the chain rule.

2. Problems

Assume that $f : [a,b] \to \mathbb{R}$ is continuous, and differentiable on (a,b). Assume that there exists exactly one point $c \in (a,b)$ where $f'(c) = 0$, and that $f(c) > 0$. Moreover, assume that $f(a) = 0 = f(b)$. Determine the absolute maximum and absolute minimum of f on $[a,b]$.

3. Problem

Assume that $f : (a,b) \to \mathbb{R}$ is differentiable on (a,b). Assume that there exists a continuous function $g : (a,b) \to \mathbb{R}$ such that $f(x) - f(y) = g(x)(x-y)$ and $g(y) = 2 - f'(y)$, for some $y \in (a,b)$. What can you say about $f'(y)$?

4. Problem

(a) Prove that $0 \leq \ln(1+x) \leq x$ for $0 < x < 1$.
(b) Determine the Taylor remainder term $R_n(x)$ in

$$\ln(1+x) = x - \frac{x^2}{2} + \cdots + (-1)^{n+1}\frac{x^n}{n} + R_n(x)\,.$$

Find an upper bound on $|R_n(x)|$ for $0 < x < 0.1$ and $n = 5$.
(c) Prove that $x - \frac{x^2}{2} \leq \ln(1+x) \leq x - \frac{x^2}{2} + \frac{x^3}{3}$ for $0 < x < 1$.

5. Problems

(a) Find $(\frac{\sin x}{e^x})'$ and $\sin(e^{x^2})'$.
(b) Find $\frac{d}{dx}\int_2^x \arctan(e^{\sin t})dt$.