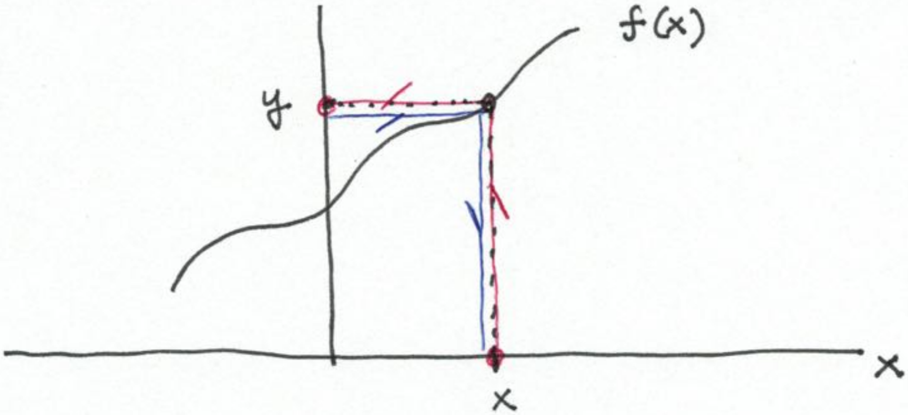
# Inverse functions.



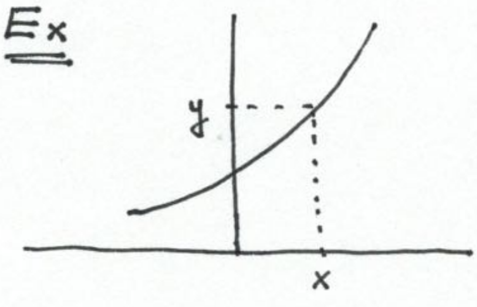

Assume $y = f(x)$ $\iff$ $x = f^{-1}(y)$ inverse fct associated to $f$.

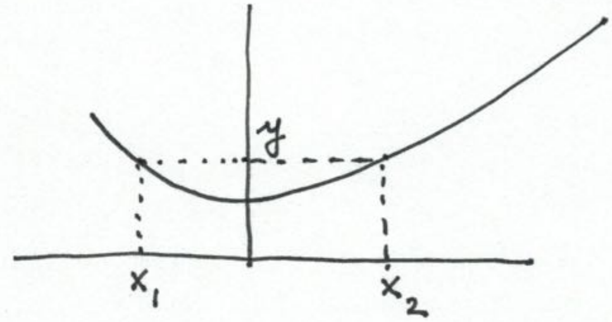
$\iff$ $f^{-1}(f(x)) = x$ , $f(f^{-1}(y)) = y$



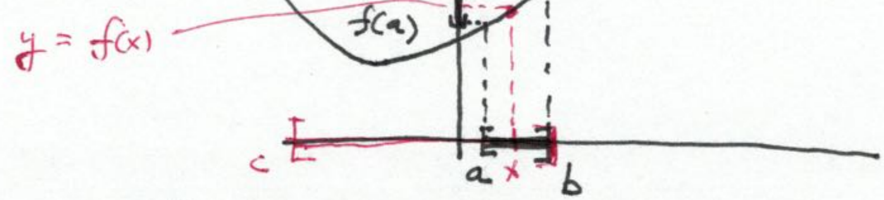
$f$ and $f^{-1}$ each undoes what the other one does.

Ex




To avoid the second case, we have to be more careful with questions concerning domain and range of $f$




look at $f$ on domain $[a, b]$

Notation: $[a, b]$ interval including $a, b$ $[a, b)$, or $(a, b]$
$(a, b)$ " excluding $a, b$

Over $[a, b]$, $f$ has precisely one $y$ value per $x$ value.
In this case, $f$ is <u>injective</u> (1-to-1) on $[a, b]$.

<u>Note</u>: $f$ <u>not</u> injective on $[c, b]$

Check for injectivity: Horizontal line test.

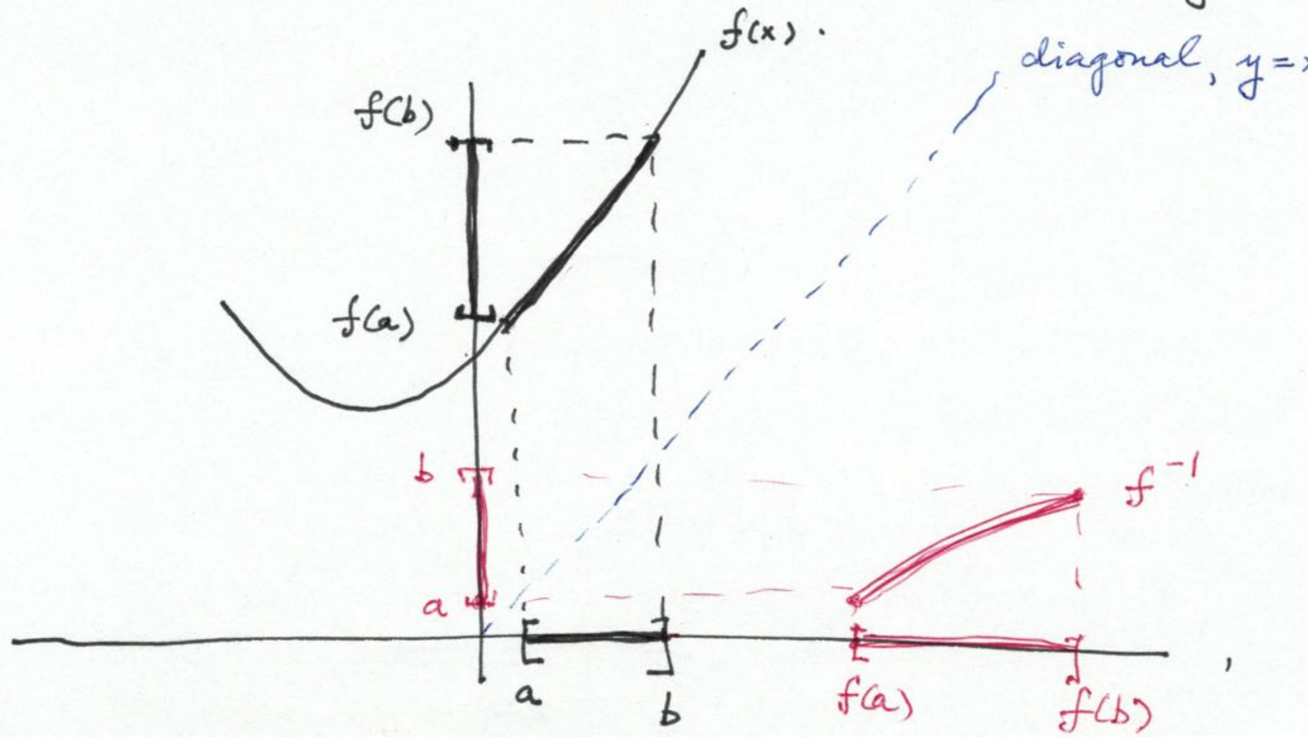
The <u>range</u> of $f$ is the set of values $f(x)$ with $x$ from the domain $[a, b]$: Here, $[f(a), f(b)]$.

The inverse fct for $f$ exists if $f$ is injective from its domain to its range.

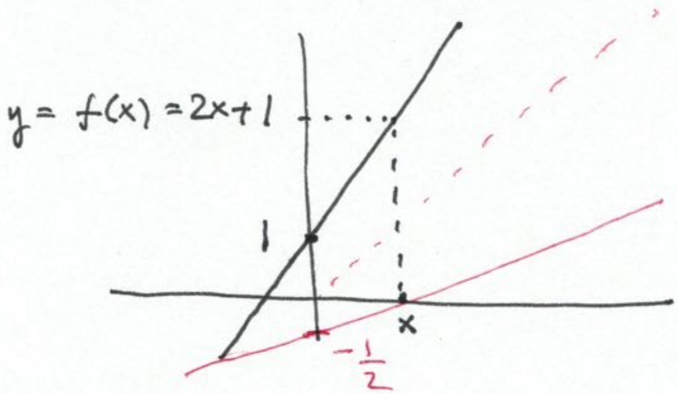
Then, the inverse fct $f^{-1}$ has as its domain $\overbrace{\text{ran}(f)}^{\text{Range of } f}$.

the " " $f^{-1}$ has as its range $\underbrace{\text{dom}(f)}_{\text{Domain of } f}$.

The graph of $f^{-1}$ is obtained from the graph of $f$ by flipping across the diagonal (exchange $x \leftrightarrow y$ axis).



**Ex**. $f(x) = 2x + 1$. , $x \in (-\infty, \infty)$




Find $f^{-1}$.

$y = 2x + 1$.

solve for $x$.

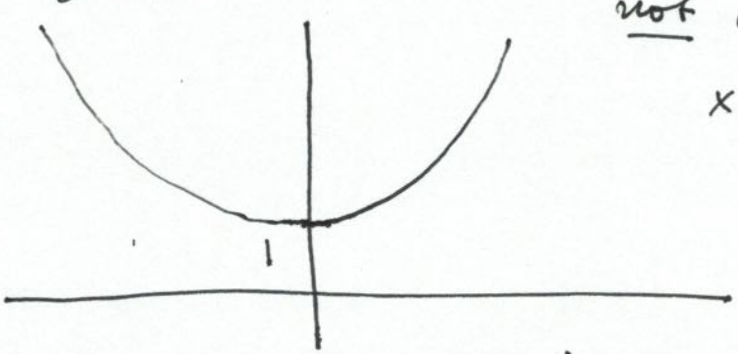
$\Rightarrow y - 1 = 2x$

$\Rightarrow x = \frac{1}{2}y - \frac{1}{2}$

flip axes: switch names $x \leftrightarrow y$.

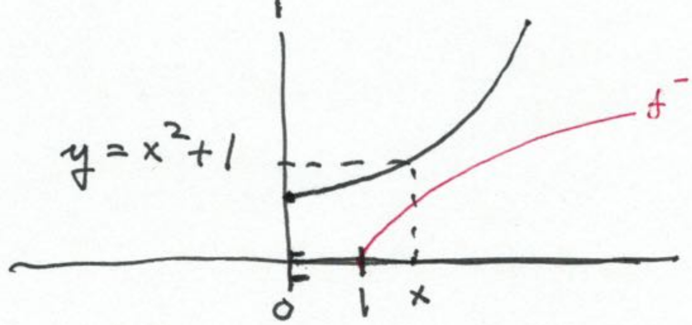
$$y = \frac{1}{2}x - \frac{1}{2}.$$

**Ex** $f(x) = x^2 + 1$.




not injective (1-to-1) for $x \in (-\infty, \infty)$.

**Ex**




injective (1-to-1) for $x \in [0, \infty)$

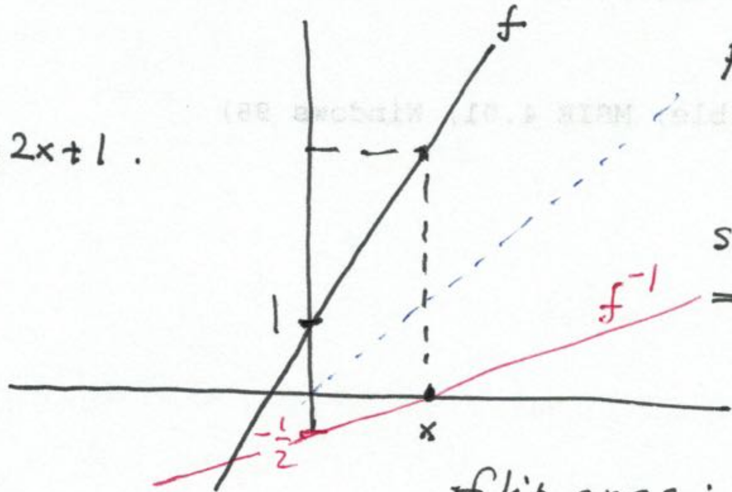
solve for $x$: $y - 1 = x^2 \Rightarrow x = \sqrt{y-1}$

switch axes: $x \leftrightarrow y$.

$$y = \sqrt{x-1} = f^{-1}(x)$$

**Ex** $\quad f(x) = 2x + 1, \quad x \in (-\infty, \infty)$

$y = 2x + 1.$




Find $f^{-1}$.

$y = 2x + 1$

solve for $x$

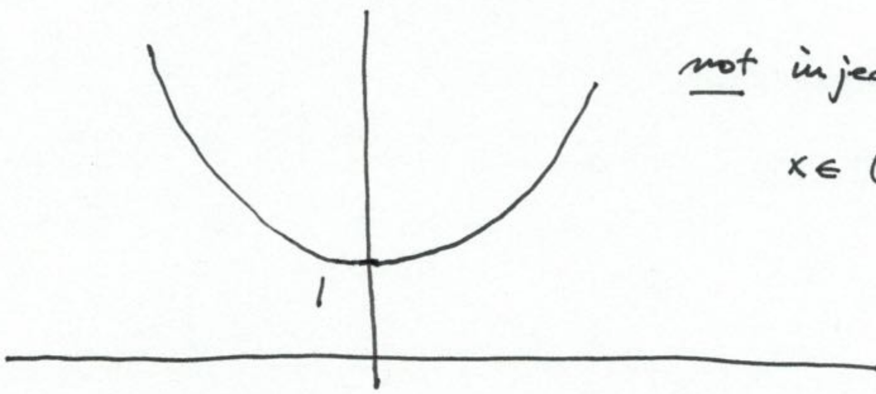
$\Rightarrow y - 1 = 2x$

$\Rightarrow x = \frac{1}{2}y - \frac{1}{2}.$

flip axes: switch names $x \leftrightarrow y$.

$y = \frac{1}{2}x - \frac{1}{2} = f^{-1}(x)$

**Ex**. $f(x) = x^2 + 1.$

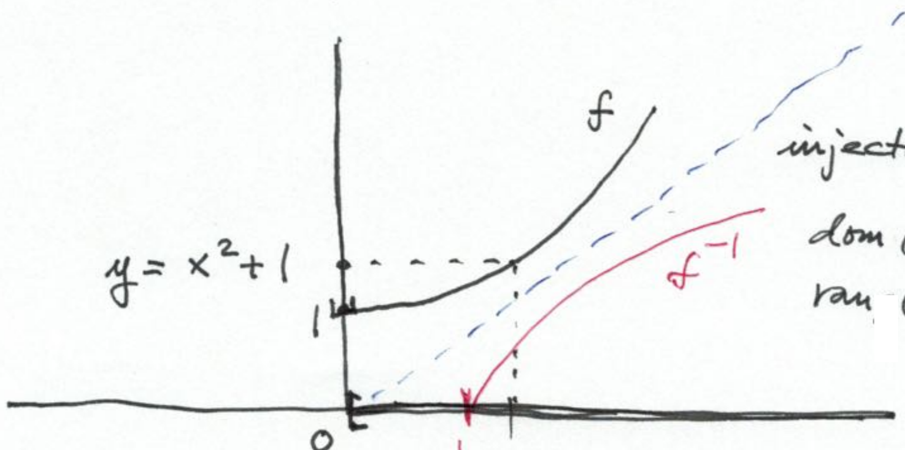



not injective (1-to-1) for

$x \in (-\infty, \infty)$.

**Ex**

$y = x^2 + 1$




injective for $x \in [0, \infty)$.

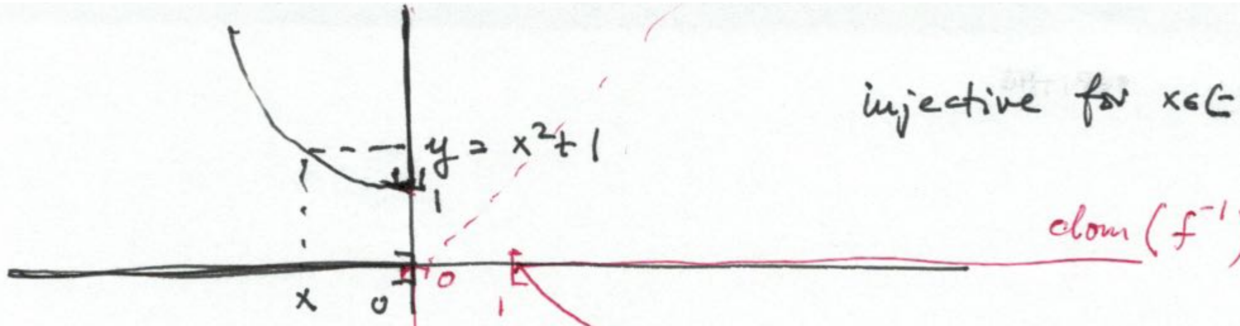
$\mathrm{dom}(f) = [0, \infty)$

$\mathrm{ran}(f) = [1, \infty)$

Solve for $x$: $\quad y - 1 = x^2 \Rightarrow x = \sqrt{y-1}$

switch axes: $x \leftrightarrow y \Rightarrow y = \sqrt{x-1} = f^{-1}(x)$

injective for $x \in [-\infty, 0]$.




solve for $x$: $y-1 = x^2 \Rightarrow x = -\sqrt{y-1}$

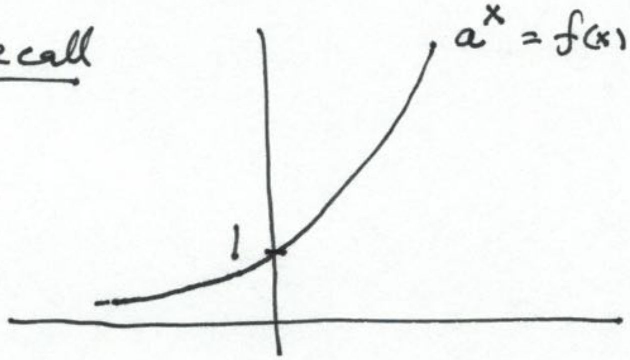
switch axes: $x \leftrightarrow y$.

$y = -\sqrt{x-1}$ , $x \in [1, \infty)$

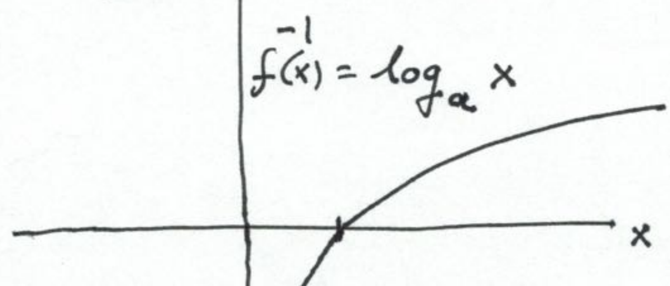
$y \in [0, -\infty)$

## Logarithms as inverse fcts of exponential fcts.

Recall




Inverse fct.




$\mathrm{dom}(f) = (-\infty, \infty)$.

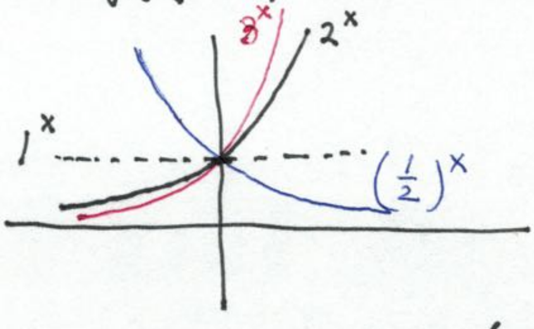
$\mathrm{range}(f) = (0, \infty)$

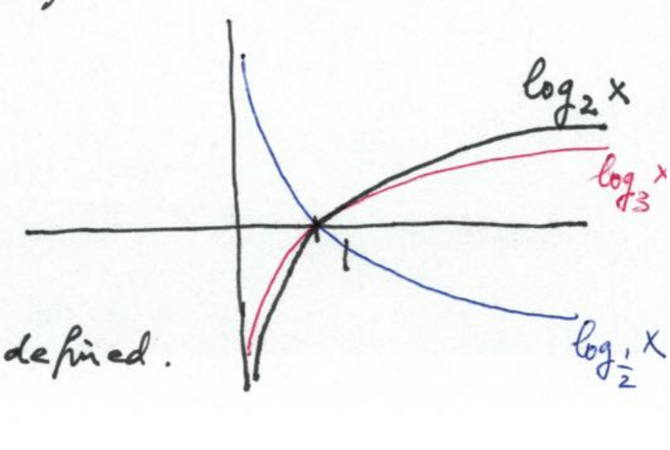
$\mathrm{dom}(\log_a) = (0, \infty)$

$\mathrm{ran}(\log_a) = (-\infty, \infty)$

$$a^{\log_a y} = f(f^{-1}(y)) = y$$

$$\log_a(a^x) = x = f^{-1}(f(x))$$




$\log_1 x$ not defined.