Left and right continuity.

Def: f is right continuous at $x=a$ if

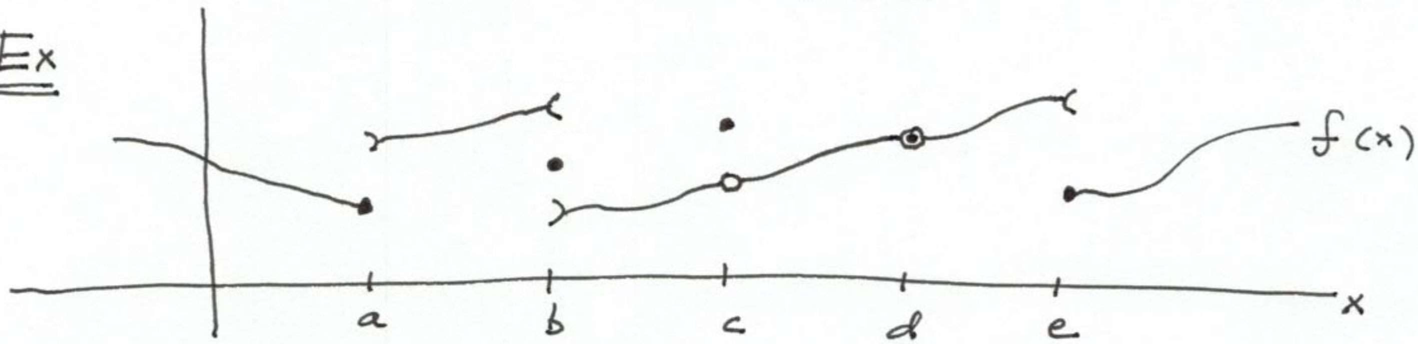
fct value at a = right limit at a.

$$f(a) = \lim_{x \to a^+} f(x).$$

f is left continuous at $x=a$ if

$$f(a) = \lim_{x \to a^-} f(x).$$

Ex

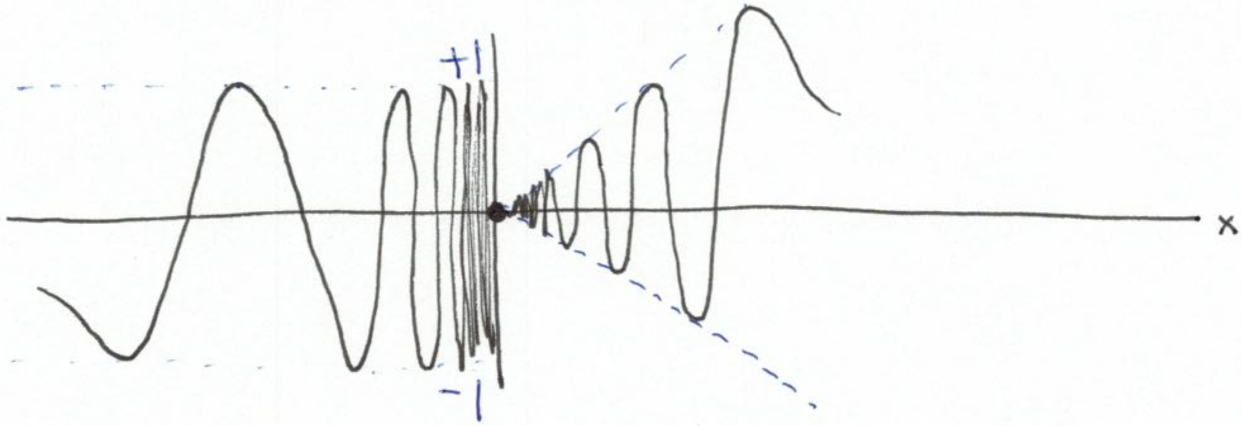


	left contin.	right contin.	continuous.
$x=a$	Y	N	N
$x=b$	N	N	N
$x=c$	N	N	N
$x=d$	Y	Y	Y
$x=e$	N	Y	N

Thm f is continuous at $x=a$ if and only if it is both left and right continuous at a.

Ex

$$f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x < 0 \\ x\sin\frac{1}{x}, & \text{if } x > 0. \\ 0, & \text{if } x = 0 \end{cases}$$



$\lim_{x\to 0^-} f(x) =$ does not exist

$\lim_{x\to 0^+} f(x) = 0 = f(0) \Rightarrow f$ is right continuous at $x=0$.

not continuous at 0.

Thm (Continuity laws).

If f and g are continuous at $x=a$, then so are

$\alpha f \pm \beta g$ (α, β numbers).

$f \cdot g$

$\frac{f}{g}$ if $g(a) \neq 0$.

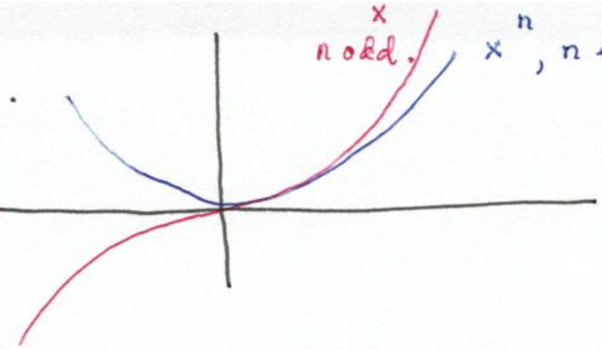
Also true for left/right continuity.

Note: f, g continuous at a implies that the function values $f(a), g(a)$ are well-defined, and thus, finite. Moreover, $\lim_{x\to a} f(x) = f(a)$ is finite, and $\lim_{x\to a} g(x) = g(a)$ is finite.

$\Rightarrow$ limits are finite, and we can use the limit laws.

Ex $f(x) = x^n$, $n \geq 0$ integer.

continuous for all x.



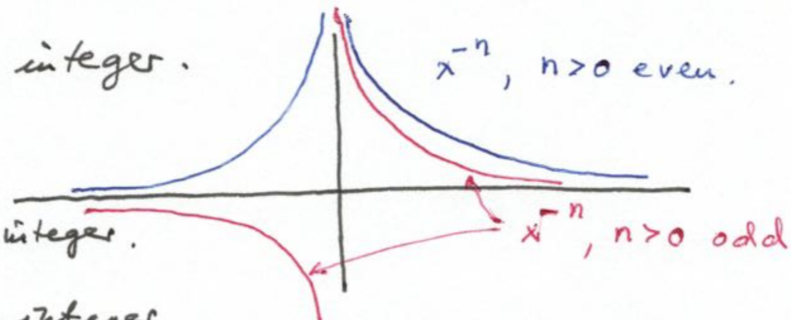
Remark: $\lim_{x \to \infty} x^n = \infty \quad n > 0$

$\lim_{x \to -\infty} x^n = \infty$, $n > 0$, even

$\lim_{x \to -\infty} x^n = -\infty$, $n > 0$, odd.

Ex $f(x) = x^{-n}$, $n \geq 0$ integer.

continuous for all $x \neq 0$



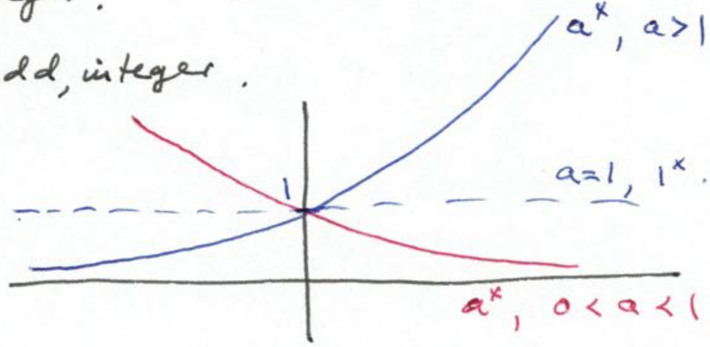
$\lim_{x \to \infty} x^{-n} = 0$, $n > 0$ integer.

$\lim_{x \to -\infty} x^{-n} = 0$, $n > 0$ integer.

$\lim_{x \to 0^-} x^{-n} = -\infty$, $n > 0$ odd, integer.

Ex $f(x) = a^x$, $a > 0$.

continuous for all x.

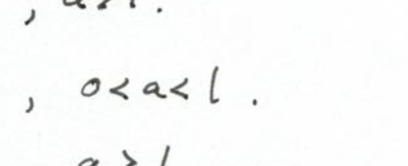


$\lim_{x \to \infty} a^x = \infty$, $a > 1$.

$\lim_{x \to -\infty} a^x = \infty$, $0 < a < 1$.

$\lim_{x \to -\infty} a^x = 0$, $a > 1$.

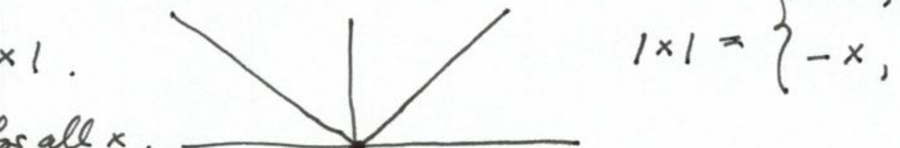
Ex $f(x) = |x|$.



$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

continuous for all x.

Ex $g(x) = |f(x)|$



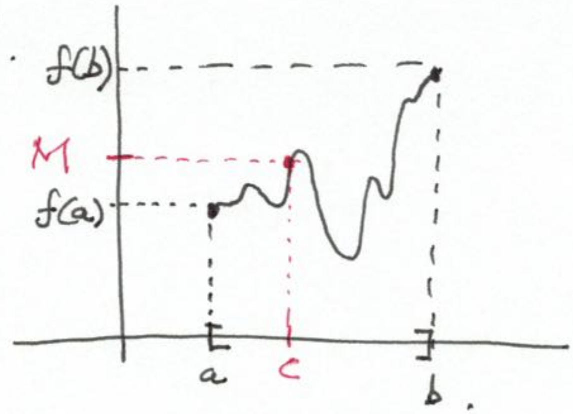
if f is continuous on (a, b), then so is $g(x) = |f(x)|$.

Thm If g is continuous at $x = a$, and f is continuous at $g(a)$, then $f(g(x))$ is continuous at a.

Ex $f(x) = \sqrt{x}$, $g(x) = e^x \Rightarrow f(g(x)) = \sqrt{e^x}$ continuous for all x.

Thm (Intermediate value theorem).

Assume f is continuous on $[a, b]$.
Then, for any number M
between $f(a)$ and $f(b)$,
there is a number c between
a and b such that $f(c) = M$.



Ex Show that the equation

$$f(x) = 4x^3 - 6x^2 + 3x - 2 = 0$$

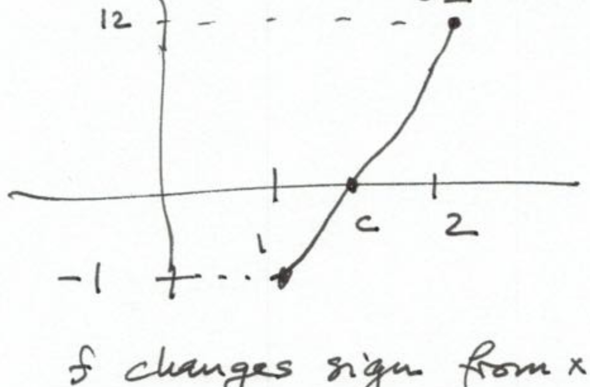
has a root between $x = 1$ and $x = 2$.

f continuous between 1 and 2.

$$f(1) = 4 \cdot 1^3 - 6 \cdot 1^2 + 3 \cdot 1 - 2 = -1$$

$$f(2) = 4 \cdot 2^3 - 6 \cdot 2^2 + 3 \cdot 2 - 2$$

$$= 32 - 24 + 6 - 2 = 12.$$



f changes sign from $x = 1$ to $x = 2$

$\Rightarrow$ there is c between 1 and 2 so that $f(c) = 0$

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