Calculus Refresher for M408D – Day 1

1. For the function defined by:

$$f(x) = \begin{cases} \dfrac{3}{x+5} & x < -1 \\ 3\left(\dfrac{x+2}{3-x}\right) & -1 < x < 0 \\ \dfrac{x^2-2x+1}{x-1} & x > 0 \end{cases}$$

Determine the following limits:

A) $\lim\limits_{x \to -1} f(x)$

B) $\lim\limits_{x \to 0} f(x)$

2. Find the limit if it exists.

A) $\lim\limits_{x \to 7} \dfrac{\sqrt{x+2}\ -3}{x-7}$

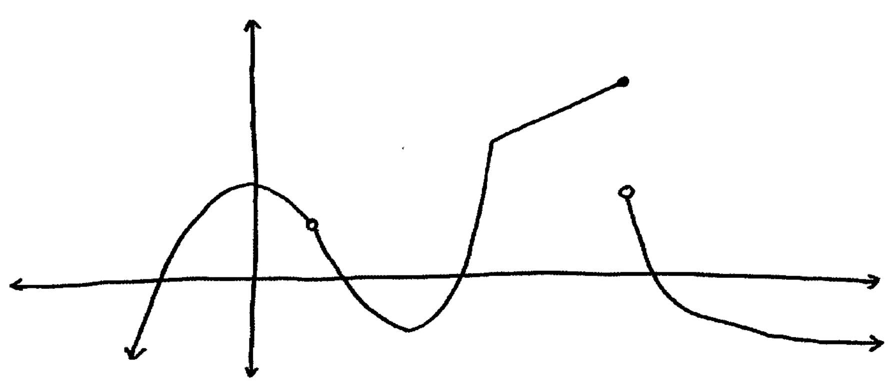
B) $\lim\limits_{t \to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right)$

C) $\lim\limits_{x \to -1} \dfrac{x^2-4x}{x^2-3x-4}$

D) $\lim\limits_{x \to 2^-} \dfrac{|x-2|}{x-2}$

3. A) State the three classifications of discontinuities, and
   B) Sketch an example of each classification

4. Draw the graph of f', given the following graph of f



5. Let

$$f(x)=\begin{cases} 4x-3 & x>1 \\ 2x^2-1 & x\le 1 \end{cases}$$

A) Is f continuous at $x=1$?

B) Is f differentiable at $x=1$?

6. Find all x-values where the tangent line to $f(x)=\frac{1}{3}x^3-x^2-13x+7\pi$ has a slope of 2.

7. Find the first derivative (dy/dx):

A) $y=\dfrac{3}{\left(2x^2+5x\right)^{3/2}}$

B) $y=\sec^2(5x)$

C) $y=\sqrt{\dfrac{1-x}{1+x^2}}$

D) $y=\ln\left(3x^2+4x\right)$

E) $y=2x-\frac{1}{2}e^{2x}$

F) $\sin\left(y^2\right)=y\cos(x)$

8. Find equation of tangent line to curve at the point (1,2)

$$x^2+2xy-y^2+x=2$$

9. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 24 cm.

10. A particle is moving along the curve $y=\sqrt{x}$. As the particle passes through the point $(4,2)$, its x-coordinate increases at a rate of 3 cm/sec. How fast is the distance from the particle to the origin changing at this instant?