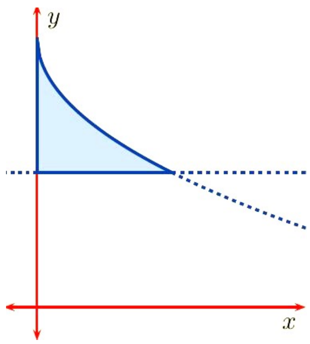
1. (5 pts) The shaded region in the figure below is bounded by the y-axis and the graphs of

$$y = 2 - \sqrt{x}, \quad y = 1.$$

Find the **volume** of the solid obtained by rotating this above region about the **x-axis**.



Solution. First solve for the intersection: $2 - \sqrt{x} = 1$, $x = 1$. The cross-sections are washer-typed.

$$\begin{aligned}
\text{Volume} &= \int_0^1 \pi \left(\left(2 - \sqrt{x}\right)^2 - 1^2 \right) dx \\
&= \pi \int_0^1 \left(4 + x - 4\sqrt{x} - 1\right) dx \\
&= \pi \int_0^1 \left(3 + x - 4\sqrt{x}\right) dx \\
&= \pi \left[3x + \frac{x^2}{2} - 4\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} \right] \Bigg|_0^1 \\
&= \pi \left(3 + \frac{1}{2} - \frac{8}{3} - 0 \right) \\
&= \frac{5}{6}\pi
\end{aligned}$$

2. (5 pts) Evaluate the integral

$$I = \int_0^1 x^2 e^x dx$$

Solution.

$$\begin{aligned}
I &= \int_0^1 x^2 e^x dx \\
&= x^2 e^x \Big|_0^1 - \int_0^1 2xe^x dx \\
&= (e - 0) - 2\int_0^1 xe^x dx \\
&= e - 2\left[xe^x \Big|_0^1 - \int_0^1 e^x dx \right] \\
&= e - 2\left(e - e^x \Big|_0^1 \right) \\
&= e - 2\left(e - (e - 1)\right) \\
&= e - 2 \cdot 1 \\
&= e - 2
\end{aligned}$$