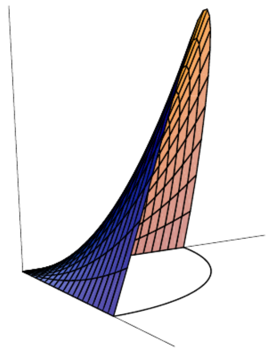
1. (5 pts) The graph of

$$f(x, y) = 5xy$$

over the bounded region R in the first quadrant enclosed by the x, y-axes and

$$y = \sqrt{4 - x^2}$$

is the surface below. Find the volume of the solid under this graph over the region R.



Solution. The volume of the solid under the graph of f is given by the double integral

$$\int\int_R f(x, y)dA$$

which in turn can be written as the repeated integral

$$\begin{aligned}
& \int_0^2 \int_0^{\sqrt{4-x^2}} f(x, y)dydx \\
= & \int_0^2 \int_0^{\sqrt{4-x^2}} 5xydydx \\
= & \int_0^2 \left(5x \cdot \frac{y^2}{2} \Big|_{y=0}^{y=\sqrt{4-x^2}} \right) dx \\
= & \int_0^2 5x \cdot \frac{(4 - x^2)}{2} dx \\
= & \int_0^2 10x - \frac{5x^3}{2} dx \\
= & 5x^2 - \frac{5}{2}\frac{x^4}{4}\Big|_0^2 \\
= & (20 - \frac{5}{2}\frac{16}{4}) - (0 - 0) \\
= & 10
\end{aligned}$$

2. (5 pts) Determine if the sequence $\{a_n\}$ converges, when

$$a_n = \cos\left(\frac{6\pi n^2(n + (-1)^n)}{18n^3 + 7n^2 + 4}\right)$$

and if it does, find its limit.

Solution.

$$\begin{aligned}
& \lim_{n\to\infty} \frac{6\pi n^2(n+(-1)^n)}{18n^3+7n^2+4} \\
= & \lim_{n\to\infty} \frac{6\pi n^3+6\pi n^2(-1)^n)}{18n^3+7n^2+4} \\
= & \lim_{n\to\infty} \frac{6\pi+6\pi\frac{1}{n}(-1)^n}{18+7\frac{1}{n}+4\frac{1}{n^3}} \text{ (devide by the highest power } n^3\text{)} \\
= & \lim_{n\to\infty} \frac{6\pi+6\pi\cdot 0}{18+7\cdot 0+4\cdot 0} \\
= & \lim_{n\to\infty} \frac{6\pi}{18} \\
= & \frac{\pi}{3}
\end{aligned}$$

Consequently, since $\cos x$ is continuous as a function of x, the sequence $\{a_n\}$ converges and has

$$\lim_{n\to\infty} \cos\left(\frac{6\pi n^2(n+(-1)^n)}{18n^3+7n^2+4}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$